# Modeling and Analyzing of Emergency Vehicle Preemption in a Four-phase Intersection via TPN

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***Abstract*—Emergency vehicles (EVs) play an important role in public service. And the EV's timeliness and safety are very important. Focusing on the switch of the phases in a four-phase intersection, this paper presents a novel phase-changing model based on timed Petri nets (TPNs) for the traffic light system and proposes a method that uses TPNs to model the preemption of emergency vehicles in such an intersection. Our model ignores many actual details. By controlling the phases, our method ensures that the EVs can pass through the intersection with no or less delay, and the safety of the traffic model on emergency scene can be ensured. To our knowledge, we are the first to employ TPNs to model EV system on a four-phase intersection. The liveness and reversibility of such a TPN model are also verified through the reachability graph of the TPN.**

*Index Terms*—**emergency vehicles, intelligent transportation systems, Petri net**

## I. Introduction

Traffic is an important part in modern society. Because of the expanding complexity of urban traffic network and the increasingly growing number of vehicles, it is imperative to improve the safety and efficiency of transportation. Now the intelligent transportation system can affect most aspects of traffic, especially of controlling the traffic signals in intersections [1]. The traffic light control systems are invented as a substitute for traffic polices for the purpose of leading pedestrians and vehicles. And the systems regulate, warn and guide transportation by allocating permission of passing in each lane during distinct time intervals. Now the traffic light control system can set up the time series of the signals according to the switch sequence of the time intervals [2].

Emergency vehicles (EVs) play an important role in serving the public as a special guardian. EVs (ambulances, fire trucks or police cars) should be able to respond to incident calls and arrive at the specific location with a minimum delay [3]. Now, by employing technologies such as sensors, wireless-network and radio, the EV can be detected and dispatched precisely.

Petri net (PN) is a powerful modeling and analysis tool for discrete event systems [4], [5]. Compared with the state machine, PN can model a system more simply, describe concurrency and the deadlock more conveniently, and has much more powerful tools to compressed representation and analysis the performance of the system [6]. For traditional PN is lack of detailed information on transitions or places, timed PN (TPN) is proposed to model the traffic network control systems [7], [8]. Moreover, timed colored Petri nets (TCPNs) are utilized as a visual formalism for modeling traffic control system [9]. In addition, PN can be used to model traffic systems on different scales, i.e., macroscopic and/or microscopic [10]. In the setting of macroscopic modeling, simple features are used to describe the behavior of the overall system [11]. But the information about individual vehicles and signals are neglected. On the other hand, microscopic traffic system models focus on individual signals, abstract zones of individual lanes and intersections [12]. And all of them pay attentions on the switch of the phases and traffic lights in some detail, but not common.

Under current traffic laws and principles, an EV often passes through an intersection very quickly with a high decibel siren, ignoring the traffic light signals. The queued vehicles in front of the EV should make way. The flow that is allowed to pass through the intersection should slow down their speed and evade the EV while hearing the siren. In the whole process, the traffic light is controlled under the original arrangement. Thus, neither the common vehicles' safety nor the EV's efficiency can be assured. Making such traffic system automatically is an important part of intelligence traffic. Huang *et al* [13] were the first to suppose an EV preemption system on intersection using TPN. However, the assumption is based on a two-phase intersection, which is rarely utilized in an urban city. Qi *et*

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*al* [14] mentioned that traffic light might change at emergency scenes and list several scenarios but took no account of the preemption of EV. Huang *et al* [8] supposed another EV preemption method based on synchronized timed Petri nets, but it ordered to stop all the traffic flow from passing the intersection, and only applied to the two-phase traffic lights system.

This paper proposes a method to model and control a four-phase intersection system for EV's preemption by using timed PN (TPN). First, we use the time information of the traffic light to model the phase-changing commonly of an intersection on a macroscopic scale, which ignore much details of the lights. Second, since the appearance and exit of an EV are stochastic but certain, another TPN model is employed to model the pass permission for each lane. We assume that every EV's coming to or leaving the intersection can be detected immediately and located accurately. Once coming is detected, the EV model is activated and affects the light model mentioned above to switch of the traffic light, which means the light system will switch the phase and give EV's lane a green light on their approach while providing red to others. The queued vehicles with different directions will be forbidden to pass, while those with the same direction as the EV are allowed to pass through the intersection without the danger of accident or being late. At the same time, the system will remain the EV's permission to pass by forbidding the phase from changing no matter how long it takes. After the EV exits, the system recovers from urgent process. To our knowledge, this is the first paper that employs TPNs to model EV system on a four-phase intersection. At the end of this paper, an example system is used to show the effectiveness of the proposed method. The model's liveness and reversibility are also verified through reachable graph.

## II. Basic Concept of TPN

A Petri net is a particular kind of bipartite directed graphs, which is comprised of two types of components, i.e. vertexes and edges. Meanwhile, the vertexes of Petri net can divide into two types, places and transitions, while the edges (arcs) of Petri net only connect places to transitions or conversely. However, edges in original Petri nets only indicate immediate transit since they cannot represent the duration of system activities [15]. Then timed Petri nets is developed to solve this problem. TPN allows three types of transitions that indicate different duration of activity: 1) an immediate one with no firing time that is represented by a thin bar; 2) a random one with an exponentially distributed delay that is represented by empty bar; 3) deterministic one with a constant delay that is represented by thick black bars. Formally, there is [16]: TPN = $(P, T, I, O, H, W, \Lambda, D, M_0)$

- $P$ is a finite set of places;
- $T$ is a finite set of transitions, which is divided into three disjoint subsets, $T_{Imm}$, $T_{det}$, and $T_{exp}$, representing immediate, deterministic and exponential ones, respectively, and, $P \cup T \neq \emptyset$ and $P \cap T = \emptyset$;
- $F \subseteq (P \times T) \cup (T \times P)$ is a set of directed arcs; $I$ means $T \times P$ while $O$ the $P \times T$;
- $H \subseteq P \times T$ is a set of inhibitor arcs from $P$ to $T$ with $H \cap F = \emptyset$;
- $W$: $(P \times T) \cup (T \times P) \rightarrow N$ is a weight function, where $W(f)>0$ if $f \in F \cup H$ and otherwise $W(f) = 0$;
- $M$: $P \rightarrow \mathrm{N}$ is a marking function, where $\forall p \in P$, $M(p)$ is the number of tokens in $p$. $M_i$ ($i$=0, 1, 2...) denotes the set of the whole $M(p)$ in the $i$th state of the system and $M_0$ denotes the initial marking;
- $\Lambda$: $T_{exp} \rightarrow R+ - \{0\}$ representing a firing rate where R+ is a set of non-negative real numbers;
- $D$: $T_{Imm} \cup T_{det} \rightarrow R+$ representing a constant firing time.

The inhibitor arc is used to link a place $p$ to a transition $t$ with a small circle attached to $t$ graphically. If an inhibitor arc with a weight of $n$, it can prevent $t$ from firing when $p$ has at least $n$ tokens. By default, $n$ is set to 1.

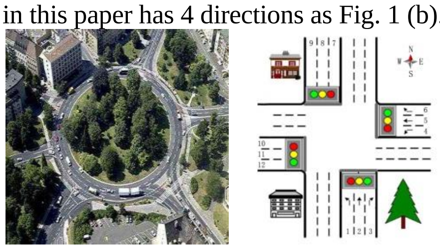
The performance of the PN model corresponds to the manifestation of the system. Some properties are very important for verifying. Two classic are deadlock-free and reversibility. A Petri net is deadlock-free if for any reachable marking from $M_0$ there is at least one transition can fire. A Petri net is reversible if that from any reachable marking it is always possible to return to the initial marking $M_0$ through appropriate transition firings. These two properties make the net safe and reusable. An effective tool for validating the properties is the reachable graph which connects each marking by the transitions enabled [13].

Conflict problem is also important in TPN when more than one transitions are enabled at the same time. An immediate transition has a higher firing priority than a timed one. In addition, if several timed transitions are enabled at the same time, the one with the shortest delay will fire.

## III. Traffic Light Control Model

### A. Intersections and Phases

Intersection is the cross-point of several roads and is one of the most important part of the traffic system. To ensure the safety of the traffic, each road is divided into several lanes leading to different directions. Two examples are indicated in Fig. 1. One is an intersection with 6 directions as Fig. 1 (a). The other example which is focuses in this paper has 4 directions as Fig. 1 (b).



(a) (b)

Figure 1. (a)An intersection with six roads. (b)An intersection with four roads.

Naturally, it turns out a method to make each lane can get permission to pass the intersection step by step. Traffic light system is used to express the permission. If several steps can comprise a fixed loop, a scheduling arrangement is generated. One step is defined as a *PHASE*.

### *B. Four-phase Intersection*

Intersections with four roads are very common in the cities of China. Such roads are often laid on by the latitude and longitude. To make the description convenient, as indicated in Fig. 1 (b), the roads lead to the north, east, south and west respectively, or N, E, S, W for short, respectively. In such a construction, many different kinds of arrangement strategy are applied to make the vehicles can pass the intersection in order.

A four-phase traffic lights system is a common arrangement, which means that the scheduling arrangement of the intersection finishes one loop and makes all the lanes get permission to pass by four steps. Each step corresponds to a set of light signals. And four steps form a cycle. Two examples are as below Fig. 2. The phase and the meaning of each strategy are described in Table I. Both the two kinds of phase circulation can be denoted in the same model to represent the change of the phase, as in Table I. For such a circulation, there is no starting phase in practical use. This model can be taken in other intersection models or automatic drive model in PN. In this paper, it is supposed that each lane can lead to only one direction and only be occupied by the vehicles in the same direction.

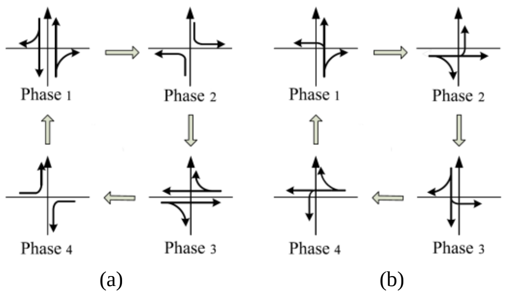



Figure 2. Two kinds of phase arrangement strategy in intersection with four roads(Four-phase traffic lights system).

TABLE I. THE PHASE AND PASSING STRATEGIES IN FOUR-PHASE TRAFFIC LIGHTS SYSTEM

| Phase | Lanes can pass in Fig 2 (a) | Lanes can pass in Fig 2 (b) |
|---|---|---|
| 1 | S to N, S to E<br>N to S, N to W | S to E, S to N, S to W |
| 2 | N to E<br>S to W | W to E, W to S, W to N |
| 3 | E to W, E to N<br>W to E, W to S | N to E, N to S, N to W |
| 4 | E to S<br>W to N | E to S, E to W, E to S |

This TPN in Fig. 3 simply models the phase-changing of the four-phase traffic lights system in a intersection. This model can be utilized in other intersection models after some extensions like [12]. The capacity of each place is 1. The place $p_{ng}$ represents that the system is in phase $n$ and the light on the lane that permits the VE to pass is green while all other lights are red. Meanwhile, $p_{ny}$ means that the system is in phase $n$ and the corresponding light is yellow while the rest lights are red. Here, $n$ can be 1, 2, 3 or 4. The deterministic transitions in the net represent the switch of the lights. In addition, $t_2$, $t_4$, $t_6$, $t_8$ represent the phase changes and the directive lights on the lane that get the permission to pass in current phase switch to red and the corresponding lights in the next phase switch to green. The transitions $t_1$, $t_3$, $t_5$, $t_7$ represent the lights are changing to yellow but the phase remains. For instrance, consider the example in Fig. 2 (a), if $t_1$ is enabled, it indicates that the lights on the lane of S to N, S to E, N to S and N to W, are green and are going to switch to yellow. If $t_2$ is enabled, it indicates that those lights are changing to red from yellow and lights on the lane of N to E and S to W are going to be green. At the same time, all the rest lights are red. The durations of each transition are set by some intersection arrangement strategy algorithm or system based on the practical situation. In most cases, the time period of yellow light (i.e., $t_2$, $t_4$, $t_6$, $t_8$) may be much shorter than that of green ones (i.e., $t_1$, $t_3$, $t_5$, $t_7$).

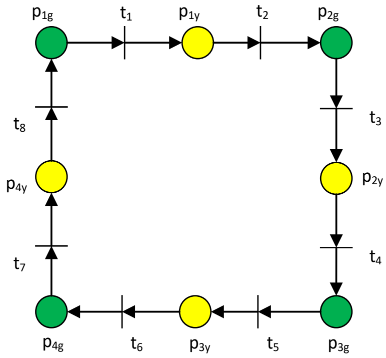



Figure 3. Petri net of the model.

Such a Petri net can model all the situations of four-phase intersections. It ignores many details of a real intersection or light system. For instance, there may be more than one lanes with the same direction in one road. By combining with reality, the four-phase light model can be used in other complex simulation of intersections. The definitions of the places and the transitions are as below. The two situations in Table II are based on Fig. 2 and Table I. Table III is based on Table II.

TABLE II. DEFINITIONS OF PLACES OF THE PETRI NET

| Place | Meaning in Figure 2 (a) | Meaning in Figure 2 (b) |
|---|---|---|
| $p_{1g}$ | Phase 1. The lights on S to N, S to E, N to S and N to W are green. The rest are red. | Phase 1. The lights on S to E, S to N and S to W are green. The rest are red. |
| $p_{1y}$ | Phase 1. The lights on S to N, S to E, N to S and N to W are yellow. The rest are red. | Phase 1. The lights on S to E, S to N and S to W are yellow. The rest are red. |
| $p_{2g}$ | Phase 2. The lights on N to E and S to W are green. The rest are red. | Phase 2. The lights on W to E, W to S and W to N are green. The rest are red. |
| $p_{2y}$ | Phase 2. The lights on N to E and S to W are yellow. The rest are red. | Phase 2. The lights on W to E, W to S and W to N are yellow. The rest are red. |
| $p_{3g}$ | Phase 3. The lights on E to W, E to N, W to E and W to S are green. The rest are red | Phase 3. The lights on N to E, N to S and N to W are green. The rest are red. |

| | | |
|---|---|---|
| $p_{3y}$ | Phase 3. The lights on E to W, E to N, W to E and W to S are yellow. The rest are red. | Phase 3. The lights on N to E, N to S and N to W are yellow. The rest are red. |
| $p_{4g}$ | Phase 4. The lights on E to S and W to N are green. The rest are red. | Phase 4. The lights on E to S, E to W and E to S are green. The rest are red. |
| $p_{4y}$ | Phase 4. The lights on E to S and W to N are yellow. The rest are red. | Phase 4. The lights on E to S, E to W and E to S are yellow. The rest are red. |

TABLE III. DEFINITIONS OF TRANSITIONS OF THE PETRI NET

| Transition | Meaning |
|---|---|
| $t_1$ | It is in phase 1, and the corresponding lights mentioned in Table II is changing to yellow from green. |
| $t_2$ | Phase 1 is switching to phase 2. |
| $t_3$ | It is in phase 2, and the corresponding lights mentioned in Table II is changing to yellow from green. |
| $t_4$ | Phase 2 is switching to phase 3. |
| $t_5$ | It is in phase 3, and the corresponding lights mentioned in Table II is changing to yellow from green. |
| $t_6$ | Phase 3 is switching to phase 4. |
| $t_7$ | It is in phase 4, and the corresponding lights mentioned in Table II is changing to yellow from green. |
| $t_8$ | Phase 4 is switching to phase 1. |

There is always only one token in this light system, which means at a certain time there is only one phase the system can be in, and only the lights directive the current phase can be green or yellow. For the purpose of initialization, one token is put in place $p_{1g}$ in the initial state, so $M_0$={1,0,0,0,0,0,0,0}. Actually, this intersection light system is not sensitive to the initial state. The reachability graph is given in Fig. 4 below:

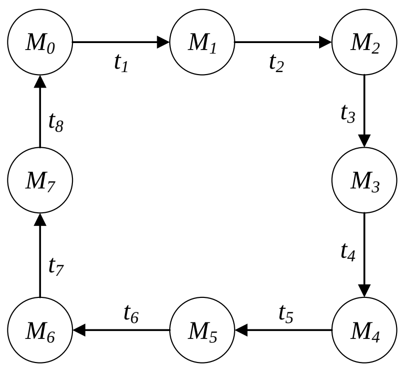



Figure 4. Reachability graph of Petri Net in Figure 3.

*C. Model of the Emergency Vehicle*

Consider the above intersection model in four-phase light system. The EV system is based on the sensors for EV. Each direction has two sensors for detecting EV, one for in and one for out. When one sensor detects the EV's drawing in, it will locate the EV's lane precisely and affect the behavior of the traffic light system, and the TPN is engaged. This system ensures that the traffic flow in the same direction as the EV can get the preemption to pass.

*1) EV on the lane that can pass in current phase*

If the EV comes from the lane that is currently allowed to pass the intersection, the system does not need to change the lights or phase so that the signal and the phase should remain. Fig. 5 indicates the situation of an EV comes from the lane which is permitted to pass because the light system is currently in the coincidental phase (here phase 1 is supposed). As Fig. 4 shows, there is only one token at the initial time. The token in $p_{1g}$ means the lights on the permitted lanes are green. When an EV going through is detected by sensors, the $t_{as}$ is enabled. The $p_{ae1}$ gets one token, which means the EV is approaching the crossing zone. The $t_{aeg}$ is enabled. Here, $t_{aeg}$ is an immediate transition with no delay, and it has priority to fire compared to $t_1$. The $p_{1g}$ got its token back and the token of $p_{ae1}$ transits to $p_{ae2}$. It hints that the light system has been interacted with by the signal of EV and the EV is going to pass the crossing zone in the stream of common vehicles.

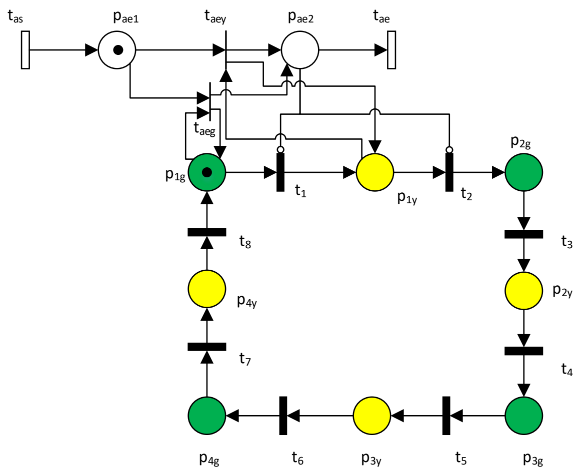



Figure 5. TPN of the case in 3.C.1.

Two inhibitor arcs are used to make sure the status of the lights will not change. One is from $p_{ae2}$ to $t_1$, the other to $t_2$. The one from $p_{ae2}$ to $t_1$ prevents $t_1$ from firing. Therefore, the lights will remain green and the EV can pass along the flow in the direction that can pass in phase 1 no matter how much time is needed. $t_{ae}$ hints the sensor just detects that the EV has passed the crossing zone. Therefore, the intersection control system must be back to normal circulation, i.e. $t_1$ can fire without competition or conflict.

In another situation, one token may in $p_{1y}$ instead of $p_{1g}$ at the beginning, which means the lights on the permitted lanes in phase 1 are yellow currently. Our strategy requests to keep the lights be yellow until the EV passes. When $p_{ae1}$ gets its token, $t_{aey}$ fires because of the priority than $t_2$. Then $p_{ae2}$ gets one token and prevents $t_2$ from firing on account of the inhibitor arc. After $t_{ae}$ fires, the token in $p_{ae2}$ is removed and the $t_2$ can be fired without conflict. It hints the EV has passed the crossing and the control system can be resumed.

In this section, phase 1 is supposed, but all the phase is suitable for this strategy. The meaning of the places and transitions is shown in Table IV.

TABLE IV. MEANING OF THE EXTRA PLACES AND TRANSITIONS IN FIG. 5 AND FIG. 6

| P/T | Meaning |
|---|---|
| $p_{ae1}$ | One EV is preparing for passing the intersection on the lane awaiting phase 1 |
| $p_{ae2}$ | EV is passing the crossing zone in the flow |
| $t_{as}$ | Sensor detects that one EV is on the lane |
| $t_{ae}$ | Sensor detects the EV is exiting from the lane |
| $t_{aeg}$ | When EV is detected and the system is in the coincidental phase and the lights on the EV's lane are green, remains the status and let the EV's flow pass |
| $t_{aey}$ | When EV is detected and the system is in the coincidental phase and the lights on the EV's lane are yellow, remains the status and let the EV's flow pass |
| $t_{4ae1}$ | When EV is detected and the lights of the different phase are green, switch to yellow immediately to cut off the passing flow and emptying the crossing zone |
| $t_{4ae2}$ | When EV is detected and the lights of the different phase are yellow, switch to phase 1 in a while |

*2) EV on the lane that cannot pass in current phase*

If the EV comes from the lane that is currently forbidden to pass the intersection, such phase must be terminated quickly and the flow with the same direction of the EV should be allowed. It means that the lights corresponding to the current phase must switch to red from green or yellow. In practice, vehicles with permission to go maybe still in the crossing zone. If the control system terminates the current phase at short notice, the probability of accident will rise. Therefore, as Fig. 6 shows, a new strategy is proposed. The meaning of places and transitions is shown in Table V. By supposing the sensor on the lane that can pass in phase 1 detects one EV, the traffic light system is affected. Here, supposes that the current phase is 4. $p_{ae1}$ gets one token in name of the EV. The $p_{4g}$ or $p_{4y}$ has one token, which hints that the lights on the lanes that can pass in phase 4 are green or yellow, respectively.

If the lights on the current phase are green, they should change to yellow first to clear the flowing vehicles in the crossing zone. $t_{4ae1}$ is an immediate transition and fires at once instead of $t_7$, the deterministic one. Then $p_{ae1}$ and $p_{4y}$ get one token respectively. It means the green lights corresponding to the current phase switch to yellow immediately to cut off the coming traffic flow and empty the crossing zone. $t_{4ae2}$ is a deterministic transition with the same delay as $t_8$. One inhibitor arc from $p_{ae1}$ to $t_8$ will forbid $t_8$ from firing, thus can forbid the phase switches automatically. After $t_{4ae2}$ fired, tokens move into $p_{1g}$ and $p_{ae2}$. It hints that the system switches to phase 1, and the lights on the lane that can get the permission to pass in phase 1 are green, including the ones on the lane of EV. Another inhibitor arc from $p_{ae2}$ to $t_1$ will forbid $t_8$ from firing until $p_{ae2}$ loses its token by $t_{ae}$. It hints that the lights on the permitted lane in phase 1 will remain green when the EV is still in the intersection. After the EV exits, the traffic system resumes from phase 1.

If the lights on the corresponding lane that can pass in current phase are yellow, the vehicles that have not entered the crossing zone on current phase will lose their priority to pass according to the traffic laws and the system's rules. The system clears the crossing zone and switches the phase to phase 1 since $t_{4ae2}$ is enabled. From now on, the process of control is as above. In addition, the traffic system resumes from phase 1 after the EV exits, too.

Here, this paper supposes the current system is in phase 4 but the EV's lane must wait for phase 1 originally. Actually, any phases are suitable for this strategy.

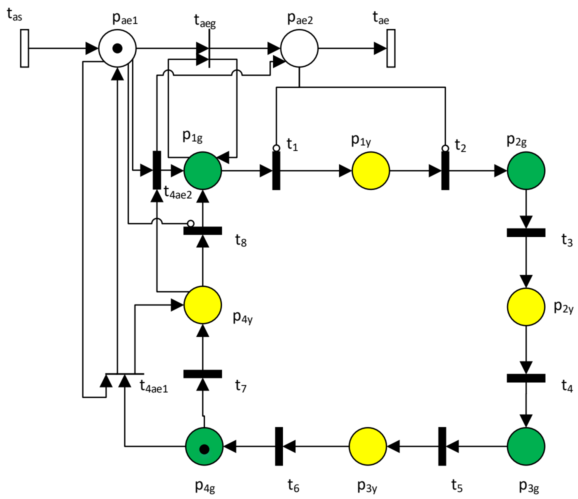



Figure 6. TPN of the case in 3.C.2.

TABLE V. MEANING OF PLACES AND TRANSITIONS IN FIG.7

| transition | meaning | type | delay |
|---|---|---|---|
| $t_{2ae1}$ | EV is detected and turn the lights to yellow from green in phase 2 immediately | immediate | - |
| $t_{2ae2}$ | Switch from phase 2 to phase 1 | deterministic | Equals to $t_4$ |
| $t_{3ae1}$ | EV detected and turn the light to yellow from green in phase 3 immediately | immediate | - |
| $t_{3ae2}$ | Turn off the current phase 3 and switch to phase 1. The EV can pass. | deterministic | Equals to $t_6$ |
| $t_{4ae1}$ | EV detected and turn the light to yellow from green in phase 4 immediately | immediate | - |
| $t_{4ae2}$ | Turn off the current phase 4 and switch to phase 1. The EV can pass. | deterministic | Equals to $t_8$ |

## IV. ANALYSIS OF THE WHOLE SYSTEM MODEL

This model needs not to restrict the relationship between current phase and the EV's direction. The whole system can be completed after copying the EV model to other 3 coming-phase besides phase 1. For brief, we only present the model with one direction for phase 1 and EV models for different phases are independent. Obviously, the system is conservative (always has limited tokens).

Fig. 7 models the brief EV model described in Section III. The meanings of places and transitions that are not given in Fig. 5 are shown in Table IV. The delay of each deterministic transition is also given.

Reachability analysis is a common method to verify the properties and correctness of Petri Nets. As this work proposed, the reachability graph of this system is based on Fig. 6 and the analysis of the different situations above.

The path of $M_0t_1M_1t_2M_2t_3M_3t_4M_4t_5M_5t_6M_6t_7M_7$ means the common switching of the traffic lights and phases. The path $M_0t_{as}M_8t_{aeg}M_9t_{ae}$ indicates the evolution of the system when an EV is detected on the lane that gets the permission to pass in phase 1 and the corresponding lights are green. Meanwhile, the path $M_1t_{as}M_{10}t_{aey}M_{11}t_{ae}$ indicates the situation when phase 1 is on yellow light. The deduction of these two paths is in 3.C.1.

The path $M_6t_{as}M_{16}t_{4ae1}M_{17}t_{4ae2}M_8t_{aeg}M_9t_{ae}$ indicates the evolution of the system when an EV is detected on the lane awaiting phase 1 but the system is in phase 4 currently. If the current lights corresponding to phase 4 are yellow, the path will be $M_7t_{as}M_{17}t_{4ae2}M_8t_{aeg}M_9t_{ae}$. For phase 2, the paths $M_2t_{as}M_{12}t_{2ae1}M_{13}t_{2ae2}M_8t_{aeg}M_9t_{ae}$ and $M_3t_{as}M_{13}t_{2ae2}M_8t_{aeg}M_9\ t_{ae}$ indicates the lights on the lane corresponding to phase 2 are green and yellow, respectively. In addition, the paths $M_4t_{as}M_{14}t_{3ae1}M_{15}t_{3ae2}M_8t_{aeg}M_9t_{ae}$ and $M_5t_{as}M_{15}t_{3ae2}M_8t_{aeg}M_9\ t_{ae}$ indicate the lights on the lane corresponding to phase 3 are green and yellow, respectively.

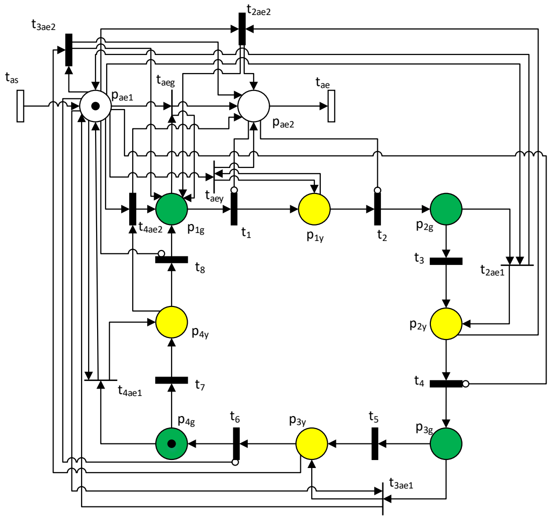



Figure 7. TPN of the whole system.

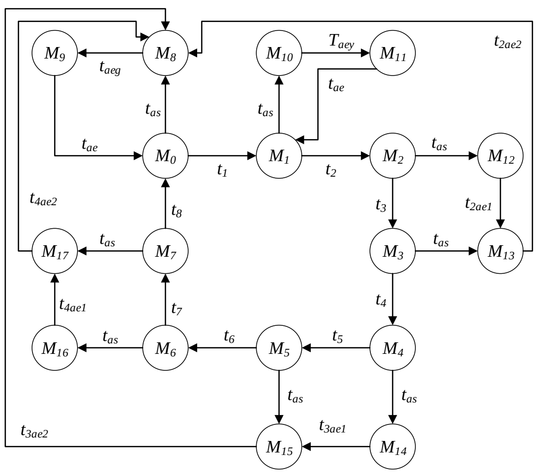



Figure 8. Reachability graph of the Petri net in Fig. 7.

From Fig. 8, we can see that for any marking reachable from $M_0$, there is at least one transition can be enabled. In addition, from any marking which is reachable from the initial marking $M_0$, there is at least one sequence of ordered transitions that make the marking $M_0$ reachable again. The reachability graph analysis shows that the model is deadlock-free and reversible. In addition, by the using of inhibitor and the priority of the transitions in Section II, this model is conflict-free.

## V. Conclusion

This work proposes a TPN model to analyze the traffic-light control system for a four-phase intersection and for emergency vehicles to avoid the traffic conflict and reduce the probability of accidents. It is very important to intelligent transportation. Our method is dedicated to a common four-phase traffic light control system, compared with the two-phase traffic light system in [13]. We use TPNs to model phase-changing and the preemption of emergency vehicles in such an intersection. The method identifies the EV first and gives the EV priority to pass and avoid traffic conflict by causing the phase of traffic light alternate. The liveness and reversibility of the proposed TPN models are verified via the reachability graph analysis. To our knowledge, this is the first work that employs TPNs to model and analyze EVs on a four-phase intersection. By slightly revising the four-phase light model, the method can be used in other complex simulation of intersections.

The approach assumes that no more than one EV appears at the same time, which may only be a simple case in practice. In addition, in another case, an EV plans to pass the intersection on a lane that can pass in phase 1 but the current phase is 3. After the system evolutions, the system needs more time to turn to phase 4 and the flow on the lane awaiting phase 4 will wait longer for their permission to pass. Therefore, we will consider the above cases in the future.

## Conflict of Interest

The authors declare no conflict of interest.

## Author Contributions

Yandong Pei and Bo Huang designed method and carried out the proof; Yandong Pei, Chunxia Zhao and Gongxuan Zhang checked the method; Bo Huang, Jiangen Hao and Yan Qiao assist to make the verification experiment. Yandong Pei and Bo Huang wrote the paper; Bo Huang, Chunxia Zhao and Yan Qiao checked the manuscript. All authors had approved the final version.

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## References

[1] N. Zhang, F.-Y. Wang, F. Zhu, and D. Zhao, "DynaCAS: Computational experiments and decision support for ITS," *IEEE Intell. Syst.*, vol. 23, no. 6, pp. 19-23, Nov.-Dec. 2008.

[2] A. Aitouche and S. Hayat, "Multiagent model using coloured Petri nets for the regulation traffic of an automated highway," in *Proc. IEEE Intelligent Transportation Systems*, 2003, pp. 37-42.

[3] H. R. Al-Zoubi, *et al*., "A wireless mobile-phone approach to traffic signal preemption for faster service of emergency vehicles," *International Journal of Computer Applications*, vol. 46, no. 3, pp. 35-41, 2012.

[4] B. Huang, M. C. Zhou, G. X. Zhang, *et al*., "Lexicographic multiobjective integer programming for optimal and structurally minimal Petri net supervisors of automated manufacturing systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 11, pp. 1459-1470, 2015.

[5] B. Huang, M. Zhou, Y. Huang, and Y. Yang, "Supervisor synthesis for FMS based on critical activity places," *IEEE Trans. Syst., Man, Cybern. Syst.*, 2018.

[6] B. Huang, M. Zhou, P. Zhang, and J. Yang, "Speedup techniques for multiobjective integer programs in designing optimal and structurally simple supervisors of AMS," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 1, pp. 77-88, Jan. 2018.

[7] L. Qi, M. Zhou, and Z. Ding, "Real-time traffic camera-light control systems for intersections subject to accidents: A petri net approach," in *Proc. IEEE Int. Conf. Syst., Man, Cybern.*, Manchester, U.K., Oct. 13-16, 2013, pp. 1069-1074.

[8] Y. S. Huang and Y. S. Weng, "Based on synchronized timed petri nets for emergency vehicle preemption systems," *Advanced Materials Research*, vol. 291, pp. 2775-2778, 2011.

[9] Y. S. An et al., "Traffic signal control model and optimization based on timed colored Petri nets." *Application Research of Computers*, vol. 4, pp. 194-198, 2016.

[10] K. M. Ng, M. B. I. Reaz, and M. A. M. Ali, "A review on the applications of Petri nets in modeling, analysis, and control of urban traffic," *IEEE Trans. Intell. Transp. Syst.*, vol. 14, no. 2, pp. 858-870, Jun. 2013.

[11] J. Júlvez and R. Boel, "A continuous Petri net approach for model predictive control of traffic systems," *IEEE Trans. Syst., Man, Cybern., A, Syst. Humans*, vol. 40, no. 4, pp. 686-697, Jul. 2010.

[12] J. Wang, J. Yan, and L. Li, "Microscopic modeling of a signalized traffic intersection using timed petri nets," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 2, pp. 305-312, 2016.

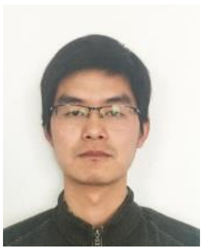
[13] Y. S. Huang, Y. S. Weng, and M. Zhou, "Design of traffic safety control systems for emergency vehicle preemption using timed petri nets," *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 4, pp. 2113-2120, 2015.

[14] L. Qi, M. C. Zhou, and W. J. Luan, "Emergency traffic-light control system design for intersections subject to accidents," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 1, pp. 170-183, 2016.

[15] L. Qi, M. C. Zhou, and W. J. Luan, "A dynamic road incident information delivery strategy to reduce urban traffic congestion," *IEEE/CAA Journal of Automatica Sinica*, vol. 5, no. 5, pp. 52-63, 2018.

[16] Y. S. Huang, Y. S. Weng, and M. C. Zhou, "Design of regulatory traffic light control systems with synchronized timed petri nets," *Asian Journal of Control*, vol. 20, no. 1, pp. 174-185, 2018.



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