Lot Sizing in Supply Chain Scheduling with Two Items Production System and Rework

Mahboubeh Ghasemi and S. Hamid Mirmohammadi
Department of Industrial and Systems Engineering, Isfahan University of Technology, Isfahan, Iran
Email: mahboobe.ghasemi@in.iut.ac.ir, h_mirmohammadi@cc.iut.ac.ir

***Abstract*—In this paper, an extended version of economic production quantity problem (EPQ) in which a final product consisted of two parts is considered. In this problem there is a dual echelon supply chain which includes the parts suppliers in first level and the final product producer in the second one. In both echelon, it is assumed that the production process accomplishes with a positive defective rate which may be different for parts and final product. In this study, production rate and rework rate is limited, demand is continuous, and shortage is not permitted. The problem is modeled by average cost approach and the optimal lot size is determined. The model is validated by illustrating numerical example.**

***Index Terms*—economic production quantity, lot sizing, supply chain management, defective production, rework**

I. Introduction

In classical models of economic lot sizing, it is assumed that all products are produced defect-free. But in practice, some percentage of produced products are diagnosed defective and therefore remanufacturing process usually is embedded in some supply chain systems. This embedment becomes more essential in the planning of supply chain systems, while more factors to be considered in addition to time and cost, e.g., environmental considerations.

There is a lot of research in recent years on the production systems with rework. (see Yoo *et al* [1] and Yoo *et al*. [2]). Ref. [3] Yassin et.al investigated the effect of defective production rate on the lot sizing policy. Ref. [4] Lee, considered an EPQ model producing definitive items. He assumed that the defective items must be reworked instantly. His study shows that the increase in defective items leads a decreas in economical batch size.

A lot of research has been done on this field. Ref. [5] Hauer and Lee *et al.* [6] considered a model with limited product rate and rework and backorder, in which decision variables were determined according to the quantity of service level. Ref. [7] Maddah and Jaber modified Jaber and Salameh's model such that they assumed the income from the sale of corrected defective items can be added to the annual profit.

In the most of studies, the quality of reworked and corrected defective products has been considered equal to the other products quality, and there is no difference for the costumers to select each one. But sometimes reworked product has different value (reworking process can reduce product quality). Therefore, some studies are conducted to show the effects of defecting and reworking on the price and customer's demand, such as Pal *et al.* [8] and Rezaei and Salemi [9]. Rezaei and Salimi considered the economic lot sizing regard to relation between the product quality and price with customer demand.

If the defective products are reworked in an independent process system and are not processed by the same sources of producing system including humans and machines, then the producing and reworking will be independent. But in this paper similar to some other research, diagnosis of defective product and rejecting them to the manufacturing system and reworking on them are processed before dispatching product to the customers. So in this situation, the quality of defected reworked product is equal to another product and a similar model is used to consider and determine the economic lot size, like Zhou *et al.* [10], Rad *et al.* [11], Widyadan *et al.* [12] and Chen and Tsao [13].

In this paper, in part two, we describe assumption condition. In part three, we formulate problem for solving it on average cost approach. Economic lot size and economic reworking in final product and segments by mathematical equation according to other parameter is calculated. In part four, we consider equation validity by numerical experiment and model analysis variable sensitivity on problem parameters (production rate, reworking rate, segment and final product defecting rate). In fifth part, we represent condition an adding up.

II. Problem Deffinition

Consider a single product manufacturing system producing a product containing two parts. In each period, Q units of this product produced to meet customer demand. Parts *a* and *b* is produced in Machines *A* and *B* then in Machine *C* they are assembled to obtain final product (see Fig. 1). To produce Q_c units of final product, Q_a units of *a* (and Q_b units of *b*) must be produced. Consumption coefficient of *a* and *b* in final product is n_a and n_b respectively, which both are integer. We assume that λ_a, λ_b and λ_c show the percent of defective items on Machine A, Machine B and Machine *C*, respectively which need to be reworked. After producing Parts *a* and *b* the inspected items which have no defect are stored and

Manuscript received April 25, 2016; revised June 22, 2016.

doi: 10.18178/jtle.4.2.178-183

they transferred to Machine C for final processing at the end of production period. After this transportation Machine *C* begins to produce the final product using Parts *a* and *b*. The produced final product is inspected immediately and defective ones are disassembled to *a* and *b* and returned to Machines A and B, respectively at the end of production period to be reworked. The perfect final products are transferred to meet costumer's demand. Manufacturing rate of each machine is limited and independent from others. Fig. 1 depicts the flow of parts and final product in this process

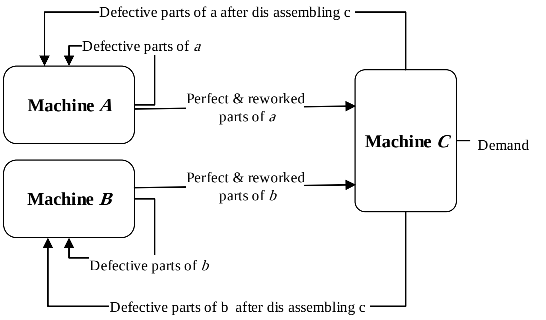

Figure 1. The production flow of the system

As many classical models in inventory management, we model the time value of money in inventory holding cost parameter, say *h*, as follows.

$$h = rc \tag{1}$$

In Eq. (1), *h* addresses holding cost of unit product, *r* discount rate which is not generally more than 0.2 and *c* the finished price of unit product. The average holding cost in general form obtained according to (2) which will be applied in the remainder of the paper.

$$C_h = \frac{1}{T}\int_0^T h(t)I(t)dt \tag{2}$$

A. Assumptions

Other assumptions of our problem are as follows.

- Inventory control policy is based on continuous review.
- Planning horizone is infinite.
- Producing rate in each machine is more than demand rate.
- There is no waste and all defective parts are reworkable.
- Final product quality is identical for customer whether the products manufactuered by primary produced parts or reworked ones.
- Reworking cost is less than producing cost for all parts.
- Inspecting and disassembly time of defective final product and the associated cost are intangible.
- Defecting rate for all parts is constant.
- Shortage is not allowed.

B. Notations

The symbols used in following equations are defined here. D: yearly customer demand rate for final product.

P_a^m, P_b^m, P_c : production rate of *a*, *b* and final product, respectively.

P_b^r, P_a^r : reworking rate of *a* and *b*.

$\lambda_a, \lambda_b, \lambda_c$: defecting rate in Machines A, B and C, respectively.

Q_b, Q_a : the lot sizes of *a* and *b* to, respectively, transfer to Machine C.

Q_b^*, Q_a^* : optimal values of Q_b, Q_a respectively.

Q_b^m, Q_a^m : produced lot sizes of *a* and *b* to, respectively.

Q_b^r, Q_a^r : reworked lot size of *a* and *b* to, respectively.

Q_c, Q_c^* : final product lot size and its optimal value respectively.

C_a^m, C_b^m, C_c : unit producing cost of *a* , *b* and final product, respectively.

C_a^r, C_b^r : unit reworking cost of *a* and *b*, respectively.

h_a^m, h_b^m, h_c : unit holding cost of *a*, *b* and final product respectively.

h_a^r, h_b^r : unit holding cost of defective *a* and *b* , respectively.

k_A, k_B, k_C : set up cost of Machines *A* , *B* and *C* respectively.

K : total annual set up cost.

$\bar{I}_a$ ، $\bar{I}_b$, $\bar{I}_c$: average inventory level of perfect Parts *a* , *b* and final product respectively.

$\bar{I}_a^r$, $\bar{I}_b^r$: average inventory level of defective Parts *a* and *b*.

t_{1a}, t_{1b} : producing period of *a* on Machine *A* and *b* on Machine *B*, respectively.

t_{2a}, t_{2b} : reworking period of *a* on Machine *A* and *b* on Machine *B*, respectively.

t_{Pc} : final product manufacturing period.

t_{dc} : final product demand period.

III. Priliminaries

To start analyzing the problem, we first consider the simpler case in which the defecting rates are zero. In this situation inventory diagram in different period for Part *a* and the final product are shown in Fig. 2. As it is shown in Fig. 2, the inventory level of *a* is decreased while producing final product by the rate of $n_a P_c$. As Shown in this figure, final product producing period (t_{pc}) is equal to consumption period t_{da} of Part *a*. These situations are also similar for Part *b*.

Customer demand of final product continuously exists and satisfying this demand with no shortage is irrevocable. For this aim, Q_c quantity of final product are produced on P_c rate. Each product is inspected and if recognized as defective product, it is disassembled to *a* and b. So in average, $Q_c^{'} = Q_c(1-\lambda_c)$ quantity of perfect final product are produced during t_{pc} and in the same time $Q_c\lambda_c$ quantity of final product are disassembled to a and b for reworking on Machines A and B, (rejected quantity to Machine A is equal to $n_a Q_c \lambda_c$ and for Machine B is $n_b Q_c \lambda_c$). After then, total defective amount of a are reworked on P_a^r rate during t_{2a} time period. This situation is depicted in Fig. 3.

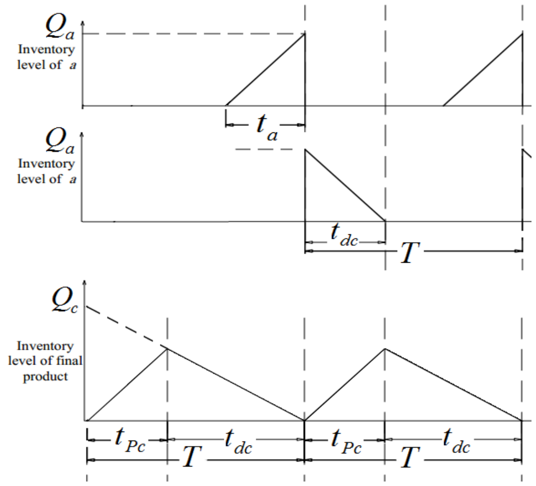

Figure 2. Inventory level of a and the final product in defect-free system

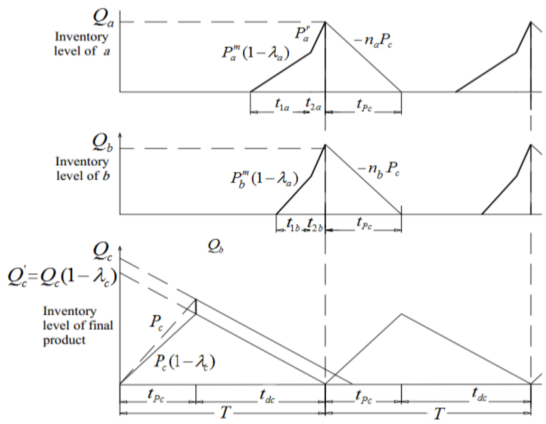

Figure 3. Inventory levels with non-zero defecting rate

As shown in the third part of Fig. 3, the inventory level of final product is depicted in both zero and non-zero defecting rate cases by two curves. The vertical distance between the curves equals $Q_c \lambda_c = Q_c - Q_c^{'}$. In each cycle T, Q_a unit of a in Machine A and Q_b unit b in Machine B must be produced. When a is producing during interval t_{1a} with P_a^m rate, defective parts are generated with the rate of λ_a , hence, at the end of producing time of Machine A we have $Q_a^m(1-\lambda_a)$ units of perfect a and $Q_a^m \lambda_a$ units of defective a. At this time defective Parts of a and rejected ones returned from Machine C are reworked during time interval t_{2a} with P_a^r rate. In other word, Machine A performs two tasks in an interval of t_a . At first part of t_a which takes time of t_{1a} , Machine A produces a from raw materials. In average Q_a^m quantity of this batch is defected. This defective parts accompanied by defective ones returned from Machine C (which is equal to $n_a Q_c \lambda_c$) are reworked in the second part of t_a which takes t_{2a} units of time. Inventory graph of part a in Machine A according to what described above is depicted in Fig. 4.

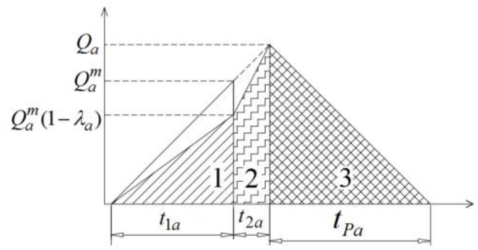

Figure 4. Inventory graph of Part a on Machine A.

We also have a same diagram for Part b. To show the level of inventory for defective parts, we show the inventory level of the defective Parts of a in Fig. 5. As it is shown in Fig. 5, defective a's are generated from producing final product (Region 1 in Fig. 5) and after then this amount increases via producing Part a whit rate $n_a Q_c \lambda_c$ by Machine A (Region 2 in Fig. 5). The inventory level of defective a is increased until Machine A begins to rework. After then, defective pasts of a is decreased to zero by rate P_a^r .

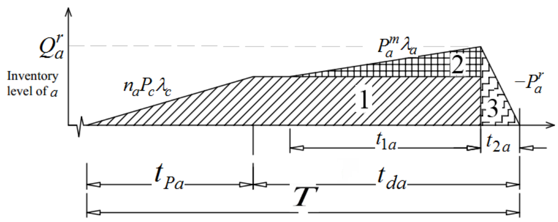

Figure 5. Inventory graph defective parts of a if $t_{dc} \geq t_a$

We depicted Fig. 5 in condition that $t_{dc} \geq t_a$, if $t_{dc} \leq t_a$ then defective inventory graph is according to Fig. 6. In this case, when production of a on Machine A begins, arrival of rejected a's from Machine C continues to take place.

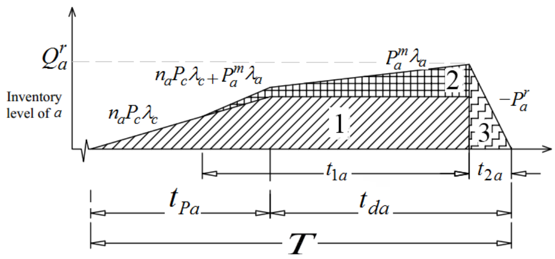

Figure. 6. Inventory graph defective parts of a if $t_{dc} \leq t_a$

IV. Problem Formulating

In this section, based on the defined notations and assumptions, we analyze the relations between several variables and we calculate the total cost function. Finally, we obtain the optimal value of decision variables via analytical approach. The equations we expressed here for Part *a* are also valid for Part *b*.

Based on what we assumed and stated previously, it is obvious that:

$$Q_a = Q_a^m(1-\lambda_a) + Q_a^r \quad (3)$$

$$Q_a^r = Q_a^m\lambda_a + n_a\lambda_c Q_c \quad (4)$$

$$Q_a = n_a Q_c \quad (5)$$

And by equation (3) to (5) we have:

$$Q_a^m = Q_a(1-\lambda_c) \quad (6)$$

For different time periods we have:

$$T = \frac{Q_c^{'}}{D} = \frac{Q_c(1-\lambda_c)}{D} \quad (7)$$

$$t_{d_c} = \frac{Q_c^{'}}{D}\left(1-\frac{D}{P_c(1-\lambda_c)}\right) = \frac{Q_c}{D}\left((1-\lambda_c)-\frac{D}{P_c}\right) \quad (8)$$

$$t_{1a} = \frac{Q_a^m}{P_a^m} \quad (9)$$

$$t_{2a} = \frac{Q_a^r}{P_a^r} \quad (10)$$

Due to problem definition and Fig. 2, the period in which the final product is being produced, Parts *a* and *b* are being consumed. Hence, we have:

$$t_{da} = t_{db} = t_{P_c} = \frac{Q_c}{P_c} = \frac{Q_a}{n_a P_c} = \frac{Q_b}{n_b P_c} \quad (11)$$

We calculate inventory cost separately in the next section.

V. Inventory Cost Systems

To calculate Q_c^* and consequently Q_a^* and Q_b^*, we need to calculate the average cost of inventory in time. Inventory system costs consists from set up cost of Machines *A* and *B* and *C* , producing and reworking cost on Parts *a* and *b* and the final product, and finally holding cost.

A. Producing and Reworking Cost

Since total annual demand quantity is fixed so the total quantity of producing and reworking quantity for Parts *a* and *b* and final product is fixed too. Therefore, producing cost of *a* and *b* and the final cost is not dependent on the quantity of decision variables and we exclude them from our further analysis.

B. Ordering Cost

According to the assumptions, set up time is intangible but there are tangible set up costs. Obviously, we have:

$$K = (k_A + k_B + k_C)\frac{1}{T} = (k_A + k_B + k_C)\frac{D}{Q_c(1-\lambda_c)} \quad (12)$$

C. Aversage Holding Cost

To calculate inventory holding cost for perfect and defective *a* and *b*, and the holding cost of the final product, we calculate average inventory of each item in Machines *A* and *B* and *C* at first.

1) Average inventory of a

To calculate the average on-hand inventory of the Part *a*, we use the regions area in Fig. 4 which are numbered by 1, 2 and 3. Let the area of these regions be S_1, S_2 and S_3, respectively. We calculate them as follows.

$$S_1 = \frac{Q_a^m(1-\lambda_a)}{2} \times t_{1a} \quad (13)$$

$$S_2 = \frac{Q_a^m(1-\lambda_a) + Q_a}{2} \times t_{2a} \quad (14)$$

$$S_3 = \frac{Q_a}{2} \times t_{Pa} \quad (15)$$

Now, for the average on-hand inventory of Part *a* we have $\bar{I}_a = \frac{S_1+S_2+S_3}{T}$.

After calculating this equation we have

$$\bar{I}_a = \frac{Q_c}{2}\varphi_a^m \quad (16)$$

where for φ_a^m we have:

$$\varphi_a^m = n_a^2 D\begin{pmatrix} \frac{(1-\lambda_c)(1-\lambda_a)}{P_a^m} + \frac{(1-\lambda_c)\lambda_a(1-\lambda_a)+\lambda_c(2-\lambda_a)+\lambda_a}{P_a^r} \\ +\frac{1}{n_a P_c(1-\lambda_c)} \end{pmatrix} \quad (17)$$

To calculate the average inventory of defective parts of *a*, we use Fig. 5. Region 1 in this figure is related to the inventory of those parts of *a* which are rejected and returned from Machine *C* over time period t_{p_c}. The area of this region is shown by $S_1^{'}$ and is calculated by Eq. (18). The second region in Fig. 5 shows the inventory diagram of the defective parts of *a* which have been generating while producing Part *a* on Machine A. The area of this region is shown by $S_2^{'}$ and is calculated by Eq. (19). Region 3 in Fig. 5 shows the inventory stream over time of overall defective parts of *a* while Machine A is reworking them with rate P_a^r. The area of this region is shown by $S_3^{'}$ and is calculated by Eq. (20).

$$S_1^{'} = \frac{n_a\lambda_c Q_c}{2} \times t_{p_c} + n_a\lambda_c Q_c(T - t_{2a}) \quad (18)$$

$$S_2^{'} = \frac{\lambda_a Q_a^m}{2} \times t_{1a} \quad (19)$$

$$S_3^{'} = \frac{Q_a^m\lambda_a + \lambda_c n_a Q_c}{2} \times t_{2a} \quad (20)$$

Since $\bar{I}_a^r = \frac{S_1^{'}+S_2^{'}+S_3^{'}}{T}$, after simplification, we have

$$\bar{I}_a^r = \frac{Q_c}{2}\varphi_a^r \quad (21)$$

in which φ_a^r is as follows.

$$\varphi_a^r = n_a^2 D\begin{pmatrix} \frac{\lambda_c}{n_a P_c(1-\lambda_c)} + \frac{2\lambda_c}{n_a D(1-\lambda_c)} \\ +\frac{\lambda_a(1-\lambda_c)}{P_a^m} + \frac{\lambda_a^2(1-\lambda_c)+\lambda_a(1-\lambda_c)+\lambda_a}{P_a^r} \end{pmatrix} \quad (22)$$

2) *Average inventory of b*

As described for Part a, average inventory for b is obtained by $\bar{I}_b = \frac{Q_c}{2}\varphi_b^m$ in which:

$$\varphi_b^m = n_b^2 D\left(\frac{(1-\lambda_c)(1-\lambda_b)}{P_b^m} + \frac{1}{n_b P_c(1-\lambda_c)} + \frac{(1-\lambda_c)\lambda_b(1-\lambda_b)+\lambda_c(2-\lambda_b)+\lambda_b}{P_b^r}\right) \quad (23)$$

and for average inventory of defective parts of *b* we have

$$\bar{I}_b^r = \frac{Q_c}{2}\varphi_b^r \quad (24)$$

where φ_b^r is as follows.

$$\varphi_b^r = n_b^2 D\begin{pmatrix} \frac{\lambda_c}{n_b P_c(1-\lambda_c)} + \frac{2\lambda_c}{n_b D(1-\lambda_c)} \\ + \frac{\lambda_b(1-\lambda_c)}{P_b^m} + \frac{\lambda_b^2(1-\lambda_c)+\lambda_b(1-\lambda_c)+\lambda_b}{P_b^r} \end{pmatrix} \quad (25)$$

3) *The average inventory of the final product*

For the average inventory of the final product, we calculate the area below the graph drawn in Fig. 3 and then we divided it to the production period length. Using related equations, we have

$$\bar{I}_c = \frac{Q_c(1-\lambda_c)}{2}\left(1-\frac{D}{P_c(1-\lambda_c)}\right) \quad (26)$$

Now, we can calculate the total average inventory holding cost. We have

$$\overline{H} = h_a^m \bar{I}_a + h_b^m \bar{I}_b + h_a^r \bar{I}_a^r + h_b^r \bar{I}_b^r + h_c \bar{I}_c \quad (27)$$

4) *Total cost of inventory system*

By Eq. (12) and (27) and replacing the average inventory terms from the related equations we obtain the total average cost of the system as follows.

$$TC = \frac{D(k_A+k_B+k_C)}{Q_c(1-\lambda_c)} + \left(\frac{Q_c h_a^m}{2}\varphi_a^m + \frac{Q_c h_b^m}{2}\varphi_b^m + \frac{Q_c h_a^r}{2}\varphi_a^r + \frac{Q_c h_b^r}{2}\varphi_b^r + h_c \bar{I}_c\right) \quad (28)$$

As it is obvious, the total cost function is the sum of to main terms which are the setup cost term and the holding cost term. The setup cost term is a decreasing function of the decision variable Q_c while the holding cost term is an increasing one. This means that the total cost function is a convex function of Q_c and this is the sufficient condition for optimality. Hence, the necessary condition for optimality is the equation $\frac{dTC}{dQ_c} = 0$. Therefore, we have

$$\frac{dTC}{dQ_c} = -\frac{D(k_A+k_B+k_C)}{Q_c^2(1-\lambda_c)} + (h_a^m\varphi_a^m + h_b^m\varphi_b^m) + \frac{1}{2}(h_a^r\varphi_a^r + h_b^r\varphi_b^r) + \frac{1}{2}\left(h_c(1-\lambda_c)\left(1-\frac{D}{P_c(1-\lambda_c)}\right)\right) = 0 \quad (29)$$

After simplifying Eq. (29), we obtain Q_c^* as follows.

$$Q_c^* = \sqrt{\frac{2D(k_A+k_B+k_C)}{(1-\lambda_c)\begin{pmatrix} h_a^m\varphi_a^m + h_b^m\varphi_b^m + h_a^r\varphi_a^r \\ + h_b^r\varphi_b^r + h_c(1-\lambda_c)\left(1-\frac{D}{P_c(1-\lambda_c)}\right) \end{pmatrix}}} \quad (30)$$

Consequently for the optimal lot sizes of Parts *a* and *b*, we have

$$Q_a^* = n_a Q_c^* \quad (31)$$

$$Q_b^* = n_b Q_c^* \quad (32)$$

Optimal quantity of Parts *a* and *b* for reworking are obtained according to equation (3) to (10) as follows.

$$Q_a^{r^*} = n_a Q_c^*(\lambda_a(1-\lambda_c)+\lambda_c) \quad (33)$$

$$Q_b^{r^*} = n_b Q_c^*(\lambda_b(1-\lambda_c)+\lambda_c) \quad (34)$$

As it can be seen in above equations, optimal quantity of Parts *a* and *b* both for producing and reworking are dependent on optimal quantity of producing final product which is produced on Machine *C*.

VI. Conclusion

In this paper, an extended version of economic production quantity problem (EPQ) in which a final product consisted of two parts was studied. The problem has been modeled via mathematical relations. Several incorporation cost of inventory systems were calculated and finally the average total cost of inventory system was obtained as a function of the final product lot size. To obtain the optimal value of the lot sizes in both echelons, the mentioned function was analyzed and the optimal values were calculated. The obtained relations showed that all optimal lot sizes are dependent on the final product lot size.

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