Abstract—The vehicle suspension system design plays an important role in the ride comfort of the passengers. In this study, the optimum design for a passive suspension system of a passenger vehicle is investigated to enhance the ride comfort for the passenger when crossing different types of speed control profiles (SCPs) while the allowable speed limit is not violated. The vehicle–passenger system is represented as a mathematical model consists of 4 degrees of freedom (4-DOF). The optimization problem aims to minimize the root mean square (RMS) of the passenger vertical acceleration subject to constraints on the maximum passenger vertical acceleration, the suspension travel, the static deflection, and the natural frequencies of both the bounce and the pitch movements. The optimization design parameters are the front suspension stiffness, the front dynamic coefficient, the rear suspension stiffness and the rear dynamic coefficient. This optimization problem is solved using the particle swarm optimization technique (PSO). Seven different speed control profiles are considered in this study to simulate and assess the vehicle performance. According to the simulation results, the optimized passive suspension system with PSO has superior performance compared with the classical passive suspension system.

Index Terms—passive suspension optimization, 4-DOF vehicle model, speed control, ride comfort, PSO

I. INTRODUCTION

The optimum design of the vehicle suspension is essential to enhance the passenger ride comfort. Different optimization problems have been studied considering passive, semi active and active vehicle suspension systems to improve the ride quality [1]-[6]. However, due to the relatively high cost and power consumed of active and semi-active suspension systems, passive ones can be used to determine proper suspension system to achieve the good ride comfort with its lowest cost. Passenger ride comfort may be defined as the level of comfort experienced by the vehicle’s passengers during travelling. Lots of studies have represented driver’s discomfort through peak vertical acceleration [3], [4], [7], while others have considered the root mean square of accelerations to quantify the human comfort [1], [2], [8], [9]. The acceptable ride comfort levels have been described in standards such as; BS 6841 [10] and ISO 2631 [11].

Speed control profiles (SCPs) have become a common passive method for controlling the speed of vehicles. The SCP should give the passenger a ride as pleasant as possible while going over a SCP below the speed limit and an unpleasant ride when speeding over the limit. The vehicle may experience different types of SCPs in one trip as seen in Fig. 1. Each SCP has a different impact on the passenger, some of them can cause uncomfortable ride even when crossing below the designed speed [12], [13]. Khorshid, Alkalby and Kamal [12], conducted an experimental investigation to evaluate the health risks associated with different geometry SCPs. Some of the tested SCPs introduced mechanical shocks to vehicle occupants beyond the health-risk zone at speeds below the specified speed limit. This means that even though the driver obeys the speed limit, there is a probability of health risks for the vehicle’s occupants. Vlastimir and Dragan studied the negative influence of vibrations to the health due to driving the typical city bus over the SCPs [13]. The results showed that passengers’ health, particularly of those using seats on rear platform, may be endangered even when crossing SCPs at speed below the speed limit. Usually the designer considers only one SCP or takes random road profile when carrying out the suspension design neglecting the effect of other SCPs the driver might cross. Therefore, an optimum design of the suspension system when crossing different types of SCPs is required.

For the Authors knowledge, the problem of passive vehicle suspension system optimization with different SCPs has not been addressed yet. In this paper, the particle swarm optimization technique (PSO) is used to optimize the vehicle passive suspension parameters of a half passenger vehicle in order to minimize the RMS of the passenger acceleration to enhance the ride comfort.

Figure 1. Different types of SCP
when crossing over different SCPs, for the first time, where constraints are imposed on some other vehicle performances to maintain them within satisfactory ranges.

II. VEHICLE MODEL

Vehicle body bounce and pitching motion are considered as the effective source of discomfort due to the resulting longitudinal acceleration [14], [15]. Accordingly, classical half vehicle model with a 4-DOF which combine both of the vehicle body pitch and the bounce motions are considered in this study [14] as seen in Fig. 2. The vehicle body mass (sprung mass), sprung mass moment, the front unsprung mass and the rear unsprung mass are identified by \( m, I, m_1, \) and \( m_2 \) respectively. The front suspension spring stiffness \( k_{sf} \) and damping coefficient \( c_{sf} \) are displaced by \( l_f \) from the vehicle center of gravity (C.G). While, the rear suspension spring stiffness \( k_{sr} \) and damping coefficient \( c_{sr} \) are displaced by \( l_r \) from C.G. \( k_{fr} \) and \( k_{rr} \) are the front and rear tires stiffness while their damping were neglected.

The bounce and pitch displacements of the vehicle are referred as \( x \) and \( \theta \) while \( x_f \) and \( x_r \) are the vertical displacements of the front and the rear unsprung masses respectively.

The road disturbances due to the SCP on the front and rear wheels are represented by the symbols \( x_{sf} \) and \( x_{sr} \) respectively. A point \( H \) is selected to define the equivalent motion of the passenger which is located at distances \( l_{fh} \) and \( l_{rh} \) from the C.G as shown in Fig. 2.

![Half vehicle model](image)

**Figure 2. Half vehicle model [14], [15]**

**TABLE I. MEDIUM SIZE VEHICLE SPECIFICATIONS**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>1350 kg</td>
</tr>
<tr>
<td>( I )</td>
<td>2400 kg.m²</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>45 kg</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>70 kg</td>
</tr>
<tr>
<td>( k_{sf} )</td>
<td>400000 N/m</td>
</tr>
<tr>
<td>( k_{sr} )</td>
<td>45000 N/m</td>
</tr>
<tr>
<td>( k_{fr} )</td>
<td>250000 N/m</td>
</tr>
<tr>
<td>( k_{rr} )</td>
<td>300000 N/m</td>
</tr>
<tr>
<td>( c_{sf} )</td>
<td>2200 N.s/m</td>
</tr>
<tr>
<td>( c_{sr} )</td>
<td>1800 N.s/m</td>
</tr>
<tr>
<td>( l_f )</td>
<td>1.21 m</td>
</tr>
<tr>
<td>( l_r )</td>
<td>1.31 m</td>
</tr>
<tr>
<td>( l_{fh} )</td>
<td>0.60 m</td>
</tr>
<tr>
<td>( l_{rh} )</td>
<td>0.30 m</td>
</tr>
</tbody>
</table>

Table I shows the vehicle specifications of a medium size passenger car which was adopted from ref. [14] to be used in the analysis. By using the concept of free-body diagram and the Newton’s second law of motion, the following equations of motion are derived;

\[
[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=[F]
\]

where,

\[
{q}=[x, \theta, x_f, x_r]^T, \quad [F]=[0, 0, k_p, x_f, k_p, x_r]^T
\]

\[
[K] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
x_{fr}(t) = x_f(t + del)
\]

where, \( del = \frac{(l_f + l_r)}{V} \).

### III. SUSPENSION OPTIMIZATION

**A. Optimization Problem**

One of the most important parts in formulating the optimization problem is the objective function. The main goal of solving the present optimization problem is to make the passenger more comfortable, when passing over different SCPs while the speed limit is not violated. Therefore, according to BS8641 [10] and ISO2631 [11], the RMS of the acceleration of the point \( H \) should be minimized to the fullest extent possible without affecting other performance characteristics of the vehicle suspension. When designing a vehicle suspension, the most concern for performance characteristics beside passenger ride comfort are; suspension travel and road holding. The suspension travel is directly related to relative distance between the unsprung mass and sprung mass while road handling is related to the tyre displacement.

Milliken *et al.* [15] gave some guidelines to be considered when vehicle suspension is designed:

- The front suspension should have a slightly lower ride rate than the rear suspension
• Pitch and bounce frequencies should be less than 1.3 Hz for passenger cars.

Therefore the optimization problem which determines the passive suspension system parameters is defined as:

Find \( W = (k_d, k_w, c_d, c_w) \)

To minimize \( f_{s0}(W) = \sum_{i=1}^{6} \text{RMS} (\ddot{x}_H) \),

Subject to \( g_1, g_2, g_3, g_4 \) where, \( i_p \) is the number of the SCPs the vehicle going to across with an allowable speed limit of 30 km/h. \( x_H \) represent the vertical movement of the passenger where, \( x_H = (x - (l_{xH}^2 + l_{zH}^2)) \times \Omega \). \( g_1 \) is constraint on the peak vertical acceleration \([16, 17]\); \( \max (\ddot{x}_H) \leq 0.5 \) gravitational acceleration. \( g_2 \) is constraint on the tyre deflection \([16, 17]\); \( |x_1 - x_{yf}| \leq 0.0508 \) and \( |x_2 - x_{yr}| \leq 0.0508 \), \( g_3 \) is constraint on the pitch and bounce frequencies that they should be \( \geq 1.3 \) Hz and the pitch frequency \( NF(2) \geq \) than the bounce frequency \( NF(1) \) [15], \( g_4 \) states that the \( k_d < k_w \) [15], and \( g_5 \) is constraint for the static deflection based on ref. [17, 18]. The static deflection \( SD \) is calculated from the following formula;

\[
SD(i) = \frac{3.133}{\sqrt{NF(i)}} \quad \text{for } i = 1 \rightarrow 4 \text{ DOF} \tag{5}
\]

The particle swarm optimization technique is applied to solve this problem. The upper and lower searching bounds for the suspension parameters used in solving the optimization problem (4) are considered to be \( \pm 50\% \) of the classical values shown in Table I. Further details for the SCPs are illustrated in the next section.

B. Particle Swarm Optimization

PSO algorithm is a computational technique originally contributed by Kennedy et al. [19], [20] which was inspired from the social behavior of the movement of organisms in a bird flock or fish school. PSO encouraged researchers from various backgrounds to use it in solving many optimization problems because it’s easy to implement, robust, fast convergence, and for its ability to solve many optimization problems [21]-[26]. PSO algorithm optimizes a problem using a population called swarm of candidate solutions called particles. Particles are initially scattered in the solution space with random initial positions and velocities. The position \( x^{(r)}_p \) and the velocity \( v^{(r)}_p \) of a particle \( \beta \) at the generation \( r \) are iteratively enhanced in the solution space towards the optimum solution. Each movement of a particle is influenced by its local position \( b^{(r)}_\beta \) and the overall best position obtained \( Ob^{(r)} \) from all the candidates in the solution space. When the process is repeated for sufficient number, the best solutions eventually will be found. Eq. (6), shown the mathematical formula used for updating the positions and the velocities of the particles [20];

\[
\begin{align*}
\alpha^{(r+1)}_\beta & = \alpha^{(r)}_\beta + v^{(r+1)}_\beta \\
v^{(r+1)}_\beta & = \chi \left( \frac{v^{(r)}_\beta + acc_i \times rand_i \times [b^{(r)}_\beta - \alpha^{(r)}_\beta]}{v^{(r)}_\beta + acc_i \times rand_i \times [Ob^{(r)}_\beta - \alpha^{(r)}_\beta]} \right) + acc_2 \times rand_2 \times [Ob^{(r)}_\beta - \alpha^{(r)}_\beta]
\end{align*}
\]

\( \chi \) is the constriction coefficient, \( acc_1 \) and \( acc_2 \) are acceleration coefficients, \( rand_i \) and \( rand_2 \) are random numbers between 0 and 1. Fig. 3 shows the optimization procedures explained above.

IV. SPEED CONTROL PROFILES

Seven SCPs were considered in this study; three parabolic and four flat-topped as shown in fig. 4 in order to achieve a comprehensive set of results. All of these profiles were selected from standards and reliable studies [27]-[29]. Table II shows the detailed dimensions of the SCPs used.
V. RESULTS AND DISCUSSIONS

The aim of this study is to increase the ride comfort of the passenger when crossing different types of SCPs under the traffic speed limit. This can be done by minimizing the $RMS$ of the passenger acceleration caused by the bounce and pitch motion of the vehicle, [10], [11], through solving the optimization problem explained in (4). A point $H$ the passenger which is located at distances $l_{th}$ and $l_{ch}$ from the C.G is considered having an equivalent motion for the driver as explained in the foregoing sections. The equations of motion for the half vehicle and the motion for the driver as explained in the forgoing sections. A point $H$ through solving the optimization problem explained in (4).

A question now arises, is it better to design the suspension when crossing multiple SCPs or is it enough to study only one single SCP. Therefore, the optimization problem is solved for two cases. The first case, it is solved when the vehicle crossing only one single SCP. And in the second case, it is solved when the vehicle independently crossing all the SCPs shown in Table II. In both cases, the vehicle is considered to travel with a speed of 30 km/h within the traffic speed limit.

A. Vehicle Crossing Single SCP

The optimization problem given in (4) is solved when the vehicle is crossing a single SCP which is SCP1 in order to obtain the optimum passive suspension parameters for minimizing the $RMS$ ($\tilde{x}_H$).

It seems from the obtained results that the $RMS$ is reduced from 1.5576 (m/s²) for the classical suspension parameters to 0.8547 (m/s²) for the optimum passive suspension parameters while crossing SCP1. The optimum suspension parameters when the vehicle is crossing single SCP will be referred as OSS. While the classical passive suspension parameters shown in Table I will be referred as CS.

B. Vehicle Crossing all SCPs

Vehicle manufacturers export their products to several countries where different type of SCPs can be found in each country. Even in the same country the vehicle could experience different type of SCPs. The suspension designer should take into account the different types of SCPs when selecting the suspension design parameters. Therefore, the optimization problem given in (4) is solved when the vehicle is independently crossing the seven different types of the SCPs detailed in Table II. The aim is to find the optimum passive suspension parameters which will minimize the overall summation of the $RMS$ $\sum_i RMS(\tilde{x}_H)$ to enhance the ride comfort of the passenger. The obtained optimum suspension parameters when the vehicle crossing all the SCPs will be referred as OAS in the hereafter. The CS replaced by the OAS and the $RMS$ values are obtained when the vehicle is crossing different SCPs. Table III shows the $RMS$ values obtained for the CS and OAS. The OAS succeeded to greatly decrease the $RMS$ values for all cases.

Fig. 5, illustrates the comparison between the $RMS$ values when using CS, OSS and OAS for different types of SCPs. Both of the OSS and OAS succeeded to greatly decrease the $RMS$ for all of the SCPs compared to CS.

Using OSS gives better responses than OAS for the case of SCP1 and SCP2 since OSS is specifically optimized when crossing the parabolic profile SCP1 which is similar to SCP2.

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>Dimensions (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parabolic LxZ</td>
<td>SCP1</td>
<td>3.65x0.1</td>
</tr>
<tr>
<td></td>
<td>SCP2</td>
<td>3.65x0.076</td>
</tr>
<tr>
<td></td>
<td>SCP3</td>
<td>2.46x0.076</td>
</tr>
<tr>
<td>Flat-top LxLxZ</td>
<td>SCP4</td>
<td>1.8x3x0.1</td>
</tr>
<tr>
<td></td>
<td>SCP5</td>
<td>2x3x0.1</td>
</tr>
<tr>
<td></td>
<td>SCP6</td>
<td>2.5x3x0.1</td>
</tr>
<tr>
<td></td>
<td>SCP7</td>
<td>1.84x3.4x0.1</td>
</tr>
</tbody>
</table>

TABLE II. TYPE SPEED CONTROL PROFILES USED

<table>
<thead>
<tr>
<th>Name</th>
<th>Dimensions (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCP1</td>
<td>1.8x3.4x0.1</td>
</tr>
<tr>
<td>SCP2</td>
<td>2x3x0.1</td>
</tr>
<tr>
<td>SCP3</td>
<td>2.53x0.1</td>
</tr>
<tr>
<td>SCP4</td>
<td>1.85x4.3x0.1</td>
</tr>
</tbody>
</table>

V. RESULTS AND DISCUSSIONS

The aim of this study is to increase the ride comfort of the passenger when crossing different types of SCPs under the traffic speed limit. This can be done by minimizing the $RMS$ of the passenger acceleration caused by the bounce and pitch motion of the vehicle, [10], [11], through solving the optimization problem explained in (4). A point $H$ the passenger which is located at distances $l_{th}$ and $l_{ch}$ from the C.G is considered having an equivalent motion for the driver as explained in the forgoing sections. The equations of motion for the half vehicle and the motion for the driver as explained in the forgoing sections. A point $H$ through solving the optimization problem explained in (4).

A question now arises, is it better to design the suspension when crossing multiple SCPs or is it enough to study only one single SCP. Therefore, the optimization problem is solved for two cases. The first case, it is solved when the vehicle crossing only one single SCP. And in the second case, it is solved when the vehicle independently crossing all the SCPs shown in Table II. In both cases, the vehicle is considered to travel with a speed of 30 km/h within the traffic speed limit.

A. Vehicle Crossing Single SCP

The optimization problem given in (4) is solved when the vehicle is crossing a single SCP which is SCP1 in order to obtain the optimum passive suspension parameters for minimizing the $RMS$ ($\tilde{x}_H$).

It seems from the obtained results that the $RMS$ is reduced from 1.5576 (m/s²) for the classical suspension parameters to 0.8547 (m/s²) for the optimum passive suspension parameters while crossing SCP1. The optimum suspension parameters when the vehicle is crossing single SCP will be referred as OSS. While the classical passive suspension parameters shown in Table I will be referred as CS.

B. Vehicle Crossing all SCPs

Vehicle manufacturers export their products to several countries where different type of SCPs can be found in each country. Even in the same country the vehicle could experience different type of SCPs. The suspension designer should take into account the different types of SCPs when selecting the suspension design parameters. Therefore, the optimization problem given in (4) is solved when the vehicle is independently crossing the seven different types of the SCPs detailed in Table II. The aim is to find the optimum passive suspension parameters which will minimize the overall summation of the $RMS$ $\sum_i RMS(\tilde{x}_H)$ to enhance the ride comfort of the passenger. The obtained optimum suspension parameters when the vehicle crossing all the SCPs will be referred as OAS in the hereafter. The CS replaced by the OAS and the $RMS$ values are obtained when the vehicle is crossing different SCPs. Table III shows the $RMS$ values obtained for the CS and OAS. The OAS succeeded to greatly decrease the $RMS$ values for all cases.

Fig. 5, illustrates the comparison between the $RMS$ values when using CS, OSS and OAS for different types of SCPs. Both of the OSS and OAS succeeded to greatly decrease the $RMS$ for all of the SCPs compared to CS.

Using OSS gives better responses than OAS for the case of SCP1 and SCP2 since OSS is specifically optimized when crossing the parabolic profile SCP1 which is similar to SCP2.

TABLE III. $RMS(\tilde{x}_H)$ VALUES FOR CS AND OAS (m/s²)

<table>
<thead>
<tr>
<th>Crossing</th>
<th>CS</th>
<th>OAS</th>
<th>Improvement %</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCP1</td>
<td>1.5576</td>
<td>0.8956</td>
<td>42.50%</td>
</tr>
<tr>
<td>SCP2</td>
<td>1.1838</td>
<td>0.6806</td>
<td>42.51%</td>
</tr>
<tr>
<td>SCP3</td>
<td>1.2081</td>
<td>0.6396</td>
<td>47.06%</td>
</tr>
<tr>
<td>SCP4</td>
<td>1.5832</td>
<td>0.8821</td>
<td>44.28%</td>
</tr>
<tr>
<td>SCP5</td>
<td>1.4977</td>
<td>0.8489</td>
<td>43.32%</td>
</tr>
<tr>
<td>SCP6</td>
<td>1.2663</td>
<td>0.7729</td>
<td>38.96%</td>
</tr>
<tr>
<td>SCP7</td>
<td>1.1523</td>
<td>0.8445</td>
<td>26.71%</td>
</tr>
</tbody>
</table>

Whereas using OAS gives better results in the Flat-top profiles (SCP4–6), it also has a better $RMS$ value than OSS for one of the parabolic profile (SCP3). From Fig. 5, it is clear that the $RMS$ values obtained for the OAS are generally better than those obtained from the OSS.

In order to assess the OAS, the vehicle dynamic performances are evaluated for the SCP1. The other SCPs (SCP2–6) have similar behaviors. Therefore they are omitted here for brevity. The time history of the half vehicle passive suspension systems behavior for the SCP1 is shown in Fig. 6.
Fig. 6-a represent the road disturbances for the front \( x_{rf} \) and rear \( x_{rr} \) for the SCP1. The passenger displacement \( x_H \) is shown in Fig. (6-b), it seems that the passenger movement and the settling time for the OAS are much lower and hence more comfortable than CS. Fig. (6-c), shows the passenger acceleration \( \ddot{x}_H \) with time. It also confirms that the passenger will be more comfortable if the OAS is used. Another point, it satisfies the constraint \( g_1 \) that the maximum acceleration is less than 0.5 of the gravitational acceleration. Figs. (6-d, 6-e) represent the tyre deflections for both the front and rear wheel. The dynamic tyre deflections are greatly decreased with OAS for both of the wheel compared to CS and consequently the dynamic tyre force decreased and the vehicle stability improved.

**VI. CONCLUSION**

This paper addressed the optimization of passive suspension parameters when crossing different speed control profiles. Seven different speed control profiles were used as the road disturbances which were adopted from road traffic standard regulations. The particle swarm optimization technique has been used to search about the optimum values of the passive suspension parameters of a half passenger vehicle. The objective of the optimization problem was increasing the ride comfort by reducing the root mean square of the passenger acceleration when constrains were imposed on other vehicle performances. The optimization problem has been solved for two cases when the vehicle crossed only single SCP and when crossed several SCPs independently. The obtained performances for both cases showed that it is generally better to optimize the suspension system when the vehicle crossing different SCPs than single SCP. The optimum values of the passive suspension parameters when crossed several SCPs were calculated to be: \( 20000 \) (N/m) for the front suspension stiffness, \( 22500 \) (N/m) for the rear suspension stiffness, \( 2504.68 \) (N.s/m) for the damping coefficient of the front suspension, and \( 1574.65 \) (N.s/m) for the rear suspension damping coefficient. The simulation results revealed that the ride comfort and vehicle stability improved effectively when using the
optimized passive suspension system with PSO compared with the classical passive suspension system.

ACKNOWLEDGMENT

This publication was supported by the European social fund within the frame work of realizing the project “Support of inter-sectoral mobility and quality enhancement of research teams at Czech Technical University in Prague”, CZ.1.07/2.3.00/30.0034. Period of the project’s realization 1.12.2012 – 30.6.2015.

REFERENCES


Ahmed Elsawaf was born in Cairo, Egypt. He obtained his M.Sc. degree in Mechanical Design Engineering from Helwan University, Egypt. He has a Ph.D. degree in Mechanical Design Engineering from Shintane University, Japan, in 2012. From 2012 to Jan. 2014, he was an Assistant Professor in the Mechanical Design Engineering Department at Helwan University, Egypt. From Feb. 2014 until now, he is a Postdoctoral Senior Researcher at the Czech Technical University in Prague, Czech Republic. His interested research areas are structural optimization, dynamic systems response, vibration control, thermal stresses suppression, modeling and identification of non-linear systems, smart materials application, and optimization of mechanical systems.

Prof. Tomáš Vampola obtained his M.Sc in applied mechanics from the CTU in Prague. After his graduate studies he worked in the Car Research Institut in Prague, where developed the computational effective algorithm for FE method for large structure. This was followed by a move to the CTU in Prague department of Mechanics where he received his Ph.D. in the field of applied mechanics. In the same department he worked as an Assistant Professor and Associate Professor. Currently he is working as a Professor in the Department of Mechanic, Biomechanics and Mechatronics, CTU in Prague. His research interest covers the modeling and simulation of large scale systems with interaction.