Bi-Level Program of Transportation Network Design Problem Accounting for Equity and Exact Solution Methodology

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***Abstract*—In the field of transportation planning, Network Design Problem (NDP) is a complex and challenging research area which aims to optimize a transportation network in order to maximize traffic and social benefits through capacity expansion and link addition. Recently, 'equity' has become highly valued in the decision process of NDP. The objective of this study is to propose novel approaches to integrate equity considerations into the NDP from the perspective of link travel time. First, equity is analysed and described mathematically in terms of travel time spent in traversing every unit-length on a link. The significance of accounting for the equity in the NDP is demonstrated through Braess Network. Then we formulate a bi-level program for NDP, where the upper level aims at optimizing system performance with respect to travel cost and equity, and the lower level is the traffic assignment problem under the user equilibrium condition. An exact solution methodology is developed based on programming techniques including interior point method, and branch and bound algorithm, which can be implemented in the general-purpose optimization software AMPL. The method can seek for the globally optimal solutions to the proposed bi-level equitable NDP model. Finally, the systematic evaluation of the developed model formulation and solution methodology is conducted on the Nguyen-Dupuis Network. The results highlight the importance of incorporating equity into transportation planning and demonstrate that the proposed approaches can generate desirable NDP decisions with respect to improving overall system performance.**

***Index Terms*—equity, network design problem, bi-level program, exact solution methodology**

I. INTRODUCTION

In transportation engineering, Network Design Problem (NDP) consists in determining which links among a transportation network should be added or expanded subject to the limited budget, in order to optimize the overall system performance [1]. In terms of model formulation, a bi-level program is most-commonly adopted for the NDP, due to its ability to describe the interaction between network planners and travellers in the decision process [2]. The lower-level program is transportation system analysis, which aims to tell us how travellers perform over the network, including travel demand and traveller's route choice behaviour [3]. It serves as the basis of the network design problem. The upper-level is an optimization problem of determining the network planning policies [4].

So far, a large number of studies on the NDP have been undertaken on model formulations and solution methodologies [5], [6]. For model formulations, most models aim at minimization of an indicator regarding travel cost. This means that travel time or congestion is the main concern and emphasis for the network design problems. As an exception, Friesz *et al*. established a multi-factors NDP model, taking both total system travel time and vehicle miles travelled into consideration [7]. Recently, incorporating equity within the NDP has received an increasing attention because of its multiple practical applications. Most studies have examined the NDP through the lens of fairness by promoting equitable capacity allocation with regards to users' origin and destination (spatial equity) [8]-[10], value of time [11] or environmental justice [12]. Typically, Chen and Yang [13] took spatial equity as a constraint in decision process of the link capacity improvement in NDP. Feng and Wu [14] applied both horizontal and vertical equities to a network design problem. Bruno Santos *et al*. [15] proposed an accessibility-maximization road network design model, considering three measures of equities in terms of accessibility of centres, accessibility to low-accessibility centres and average accessibility of subregions. These representations of equity can be approached using a multi-class NDP model, where users are categorized by classes and where equity across these classes is sought. These studies focused on the spatial equity across zone areas, where a gap exists that zone division patterns have inevitable influences on the planning policies.

For solution methodologies, since a bi-level program of NDP is a non-convex and NP-hard problem [16], [17], most studies have focused on meta-heuristic algorithms, including simulated annealing algorithms [18], neural network algorithms [19], genetic algorithms [20], ant colony optimization algorithms [21], hybrid meta-heuristic algorithms [22], particle swarm optimization [23]. By contrast, few exact solution methodologies have been developed in previous studies [24], [25]. Although some traditional exact methods such as the branch and bound algorithm, branch-backtrack, bender

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decomposition can solve the bi-level NDP models to the optimality in theory, most of them are not applicable or transferrable due to computational inefficiency [26]. However, exact solution methodologies are of vital importance to the mathematical optimization, since it can provide us with the globally optimal solutions to the developed models. Moreover, the advent of new technologies can promote the development of exact solution methodologies by the increase in computational power.

The objectives of this study are to: 1) propose a novel bi-level model formulation to integrate equity in terms of link travel time into NDPs; 2) develop an exact solution methodology to the equitable NDP model; and 3) evaluate efficacy of the proposed approaches over a synthesized test network. We seek to find the optimal capacity allocation such that the overall travel cost and the equity are optimized. This rest of this paper is organized as follows. In Section 2, equity in a transportation network is described mathematically and demonstrated over Braess Network. In Section 3, the equitable NDP is formulated into a bi-level programming model. In Section 4, an exact solution methodology is developed, based on techniques including interior point method, branch and bound algorithm. In Section 5, evaluation of the proposed model formulation and solution methodology is conducted on the Nguyen-Dupuis Network. Section 6 makes conclusions and presents future studies.

II. Motivation

In this section, we discuss the equity in a transportation network and present an indicator of equity. Then the significance of accounting for equity in the NDP is demonstrated upon Braess Network.

A. *Choice of Equity Measure*

There exist a number of equity metrics considered in NDPs. Typically, zonal equity stems from the point that the network planners seek to balance travel costs from different zones to zones. Other kinds of equity could primarily depend on the link travel time, which this study will concentrate on. One measure of the equity could be proposed based on the point that the absolute equity in an ideal condition should be achieved if travel time spent on any unit length of any lane over the network remains the same. Under this circumstance, all the network users can be expected to travel at the same speed regardless of their selected routes and links, with their travel time strictly in direct proportion to the traversed distance. Then the measure of equity is to use the dispersion of travel time spent in traversing every unit length among the network. Mathematically, the dispersion of unit-length travel time can be indicated by the standard deviation, which is defined as follows:

$$SD(\tau) = \left[E\left[\tau_{ij} - E(\tau)\right]^2\right]^{\frac{1}{2}} \tag{1}$$

where τ and τ_{ij} represent the unit-length travel time and that on link (i,j) respectively. $SD(\cdot)$ and $E(\cdot)$ are notations of standard deviation and expectation respectively. The standard deviation of unit-length travel time $SD(\tau)$ can be calculated as follows:

$$\begin{aligned} SD(\tau) &= \left[E\left[\tau_{ij} - E(\tau)\right]^2\right]^{\frac{1}{2}} \\ &= \left[\sum_{(i.j)\in A} P_{ij}\cdot\left[\tau_{ij} - E(\tau)\right]^2\right]^{\frac{1}{2}} \\ &= \left[\sum_{(i.j)\in A} P_{ij}\cdot\left[\tau_{ij} - \sum_{(i.j)\in A}\left(P_{ij}\cdot\tau_{ij}\right)\right]^2\right]^{\frac{1}{2}} \end{aligned} \tag{2}$$

where

$$\tau_{ij} = \frac{t_{ij}(x_{ij}, y_{ij})}{L_{ij}} \tag{3}$$

$$P_{ij} = \frac{L_{ij}}{\sum_{(i,j)\in A} L_{ij}} = \frac{L_{ij}}{L} \tag{4}$$

By inserting Equations (3) and (4) into Formula (2),

$$SD(\tau) = \left[\sum_{(i.j)\in A}\frac{L_{ij}}{L}\cdot\left[\frac{t_{ij}(x_{ij}, y_{ij})}{L_{ij}} - \sum_{(i.j)\in A}\left(\frac{L_{ij}}{L}\cdot\frac{t_{ij}(x_{ij}, y_{ij})}{L_{ij}}\right)\right]^2\right]^{\frac{1}{2}}$$

$$= \left[\sum_{(i.j)\in A}\frac{L_{ij}}{L}\cdot\left[\frac{t_{ij}(x_{ij}, y_{ij})}{L_{ij}} - \frac{\sum_{(i,j)\in A} t_{ij}(x_{ij}, y_{ij})}{L}\right]^2\right]^{\frac{1}{2}} \tag{5}$$

The greater the standard deviation is, the more dispersedly the unit-length travel time is distributed and consequently, less equity the planning policy has achieved. On the other hand, the absolute equity is achieved if the value of $SD(\tau)$ equals zero. It is this measure that we will use in our model formulation.

B. *Motivating Example*

In this section, the importance of accounting for equity in network design problems is demonstrated via the Braess Network shown in Fig. 1. Link parameters used for the network are listed in Table I. For origin-destination (OD) travel demands, there are six vehicle-trips from Node 1 to Node 4 in this network.

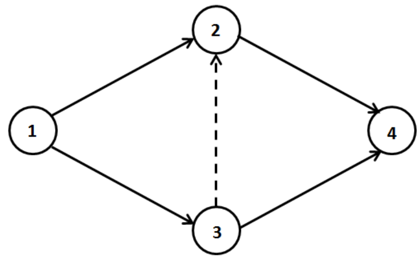

Figure 1. Braess network

TABLE I. Link Parameters for Braess Network

No.	Link	Length	Link Travel Time Function
1	(1,2)	2	$(50 + x)$*
2	(1,3)	2	$10x$
3	(2,4)	2	$10x$
4	(3,4)	2	$(50 + x)$
5	(3,2)	2	$(10 + x)$
*x refers to link flow.			

Initially, Link (3,2) doesn't exist in the network. Then the network is modified by adding the Link (3,2). After modification, the travel demand has been re-assigned to the network under the user equilibrium (UE) condition.

The traffic assignment results and system performance on both initial and modified networks are summarized in Table II.

TABLE II. TRAFFIC PERFORMANCE ON INITIAL AND MODIFIED NETWORKS

Initial Network				
No.	Link	Link Flow	Link Travel Time	Unit-length Travel Time
1	(1,2)	3	**53**	26.5
2	(1,3)	3	**53**	26.5
3	(2,4)	3	**30**	15
4	(3,4)	3	**30**	15
Total System Travel Time			498	
Standard Deviation of Unit-length Travel Time			5.750	
Modified Network				
No.	Link	Link Flow	Link Travel Time	Unit-length Travel Time
1	(1,2)	2	**52**	26
2	(1,3)	4	**40**	20
3	(2,4)	4	**40**	20
4	(3,4)	2	**52**	26
5	(3,2)	2	**12**	6
Total System Travel Time			552	
Standard Deviation of Unit-length Travel Time			7.310	

TABLE III. NOTATIONS

$\boldsymbol{N}$	Node set
$\boldsymbol{A}$	Link set
$\boldsymbol{Z^2}$	OD pair set: $\boldsymbol{Z^2} \subseteq \boldsymbol{N} \times \boldsymbol{N}$
$\boldsymbol{\Pi}$	Set of paths
$\boldsymbol{\Pi^{rs}}$	Set of paths for OD pair $(\boldsymbol{r}, \boldsymbol{s}) \in \boldsymbol{Z^2}$
$(\boldsymbol{i}.\boldsymbol{j})$	Link with upstream node $\boldsymbol{i} \in \boldsymbol{N}$ and downstream node $\boldsymbol{j} \in \boldsymbol{N}$
$\boldsymbol{c_{ij}}$	Capacity of link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$
$\boldsymbol{x_{ij}}$	Flow on link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$, $\boldsymbol{x_{ij}} \in \boldsymbol{x}$
$\boldsymbol{h^{\pi}}$	Flow on path $\boldsymbol{\pi} \in \boldsymbol{\Pi}$
$\boldsymbol{t_{ij}}$	Travel time on link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$
$\boldsymbol{t^0_{ij}}$	Free flow travel time on link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$
$\boldsymbol{d^{rs}}$	Travel Demand for OD pair $(\boldsymbol{r}, \boldsymbol{s}) \in \boldsymbol{Z^2}$
$\boldsymbol{y_{ij}}$	The degree to which link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$ is added or improved, $\boldsymbol{y_{ij}} \in \boldsymbol{y}$
$\boldsymbol{\gamma_{ij}}$	The unit expenditure of addition or improvement for link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$
$\boldsymbol{B}$	Total budget available for network design
$\boldsymbol{\tau_{ij}}$	Travel time per unit-length on link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$
$\boldsymbol{L_{ij}}$	Length of link $(\boldsymbol{i}, \boldsymbol{j}) \in \boldsymbol{A}$
$\boldsymbol{L}$	Sum of lengths of all the links among the network
$\boldsymbol{TSTT}$	Total system travel time

Table II shows that after modification, both total system travel time and standard deviation of unit-length travel time have increased. It means that adding Link (3,2) doesn't lead to improvement in the traffic performance on the network. On the contrary, traffic performance deteriorates in terms of both travel cost and equity. The results are in accordance with the Braess Paradox, which states that since travellers choose their routes 'selfishly', adding extra capacity to a network might sometimes reduce overall system performance. In most cases, the overall system performance just refers to the total system travel time, but in the example above, equity with respect to unit-length travel time is also incorporated. Therefore, in order to improve network performance, it is essential to develop a mathematical programming model of NDP accounting for both travel cost and equity.

III. MODEL FORMULATION

A. Notations

Notations used throughout the paper are listed in Table III unless otherwise specified.

B. Bi-Level Program of Equitable NDP

In terms of the equity approach proposed in Section 2, the time spent in traversing any unit length of any link in the whole network would be the same when the absolute equity is achieved. By integrating the equity into NDP, the objective of the planning policy is to minimize the weighted sum of the travel cost term I_c and equity term I_e among the network, *i.e.*

$$\min_y \ \alpha \cdot I_c + (1-\alpha) \cdot I_e \tag{6}$$

where travel cost term I_c is indicated by total system travel time, $TSTT(\boldsymbol{x}, \boldsymbol{y})$; equity term I_e is derived from the standard deviation of unit-length travel time $SD(\tau)$ adjusted by the factor of network-scale impact $f(\boldsymbol{x}, \boldsymbol{y})$, taking network scales into consideration; α is the weighting factor regarding travel cost versus equity, ranging from zero to one, which can be determined by planners' preference. Thus, the objective function can be specified as follows:

$$\min_y \ \alpha \cdot TSTT(\boldsymbol{x}, \boldsymbol{y}) + (1-\alpha) \cdot f(\boldsymbol{x}, \boldsymbol{y}) \cdot SD(\tau) \tag{7}$$

In this objective function, $SD(\tau)$ is defined as general equity term and $f(\boldsymbol{x}, \boldsymbol{y}) \cdot SD(\tau)$ as network-scale-adjusted equity term, or adjusted equity term for short. The values of both equity terms need to be lowered, so as to make the planning policy more equitable.

Therefore, the bi-level programming model for network design problem accounting for equity can be formulated as follows:

Upper-level program (U):

$$\min_y \ \alpha \cdot \sum_{(i,j)\epsilon A} x_{ij} \cdot t_{ij}\left(x_{ij}, y_{ij}\right) + (1-\alpha) \cdot f(\boldsymbol{x}, \boldsymbol{y}) \cdot SD(\tau) \tag{8}$$

subject to

$$\textstyle\sum_{(i,j)\in A} \gamma_{ij} \cdot y_{ij} \leq B \tag{9}$$

$$y_{ij} = 0,1,2 \ldots \quad \forall (i,j) \in A \tag{10}$$

where the flow pattern $\boldsymbol{x}$ can be obtained by solving the traffic assignment problem below:

Lower-level program (L):

$$\min_{\mathrm{x,h}} \ \textstyle\sum_{(i,j)\in \mathrm{A}} \int_0^{x_{ij}} t_{ij}\left(x, y_{ij}\right) dx \tag{11}$$

subject to

$$x_{ij} = \textstyle\sum_{\pi \in \Pi} \delta_{ij}^{\pi} \cdot h^{\pi} \qquad \forall (i,j) \in A \tag{12}$$

$$d^{rs} = \textstyle\sum_{\pi \in \Pi^{rs}} h^{\pi} \qquad \forall (r,s) \in Z^2 \tag{13}$$

$$h^{\pi} \geq 0 \qquad \forall \pi \in \Pi \tag{14}$$

The model formulation comprises two levels. The upper level is the transportation planning problem associated to measures of link addition and capacity expansion, with planners' objective of optimizing system performance in travel cost and equity. The lower level is the traffic assignment problem under the UE condition. In the upper level, Formulation (8) is the objective function of NDP; Constraint (9) is about the limitation of budget; and Constraint (10) specifies the feasible region of design decision variables. In the lower level, Formulation (11) is the objective function of UE traffic assignment model; Constraint (12) describes conservation relationship between link and path flows; Constraint (13) describes conservation relationship between path flows and OD demands; and Constraint (14) is about non-negativity of path flows.

Two terms in the bi-level model are discussed as follows:

1) Adjustment factor of network-scale impact ($f(\boldsymbol{x},\boldsymbol{y})$):

In the model, the general equity term $SD(\tau)$ indicates the dispersion of unit-length travel time among the network. It does not have direct connection to the network scale. By contrast, the term $TSTT$ in the model indicates the sum of travel time for the network and tends to become far greater in a larger network under higher travel demands. In this case, as compared to $TSTT$, the $SD(\tau)$ is too small to account for, especially for the large-scale network. Even if the network is small-scale, the difference between $SD(\tau)$ and $TSTT$ might be striking (e.g. for Braess Network in Section 2, $SD(\tau)$ 5.750 versus $TSTT$ 498 for the initial network, 7.310 versus 552 for the modified network). In order to balance the terms of travel cost and equity in the objective function, the equity term for every specific network should be formulated by $SD(\tau)$ multiplied by the adjustment factor of network-scale impact $f(\boldsymbol{x},\boldsymbol{y})$. The value of $f(\boldsymbol{x},\boldsymbol{y})$ can be determined based on how $TSTT$ reflects the network-scale impact:

$$TSTT = \sum_{(i,j)\epsilon A} t_{ij} \cdot x_{ij} = \sum_{(i,j)\epsilon A} \tau_{ij} \cdot L_{ij} \cdot x_{ij} \quad (15)$$

It shows that in travel cost term, $TSTT$ actually equals the sum of unit-length travel time of every lane (τ_{ij}) multiplied by length times flow. Similarly, the network-scale-adjusted equity term I_e should be formulated as the sum of standard deviation of unit-length travel time ($SD(\tau)$) times length and flow, *i.e.*

$$I_e = f(\boldsymbol{x},\boldsymbol{y}) \cdot SD(\tau) = \sum_{(i,j)\epsilon A} SD(\tau) \cdot L_{ij\cdot} \cdot x_{ij} \quad (16)$$

Since $SD(\tau)$ is a constant for a certain flow pattern, the adjusted equity term can be expressed by

$$I_e = SD(\tau) \cdot \sum_{(i,j)\epsilon A} L_{ij} \cdot x_{ij} \quad (17)$$

Therefore, the adjustment factor of network-scale impact should be:

$$f(\boldsymbol{x},\boldsymbol{y}) = \sum_{(i,j)\epsilon A} L_{ij} \cdot x_{ij} \quad (18)$$

2) Link travel time $t_{ij}(x_{ij},y_{ij})$:

Link travel time $t_{ij}(x_{ij},y_{ij})$ is significantly related to the link flow and capacity. It can be estimated by link performance functions, among which the BPR function is the most-commonly used one [27]. Based on BPR function, the link travel time $t_{ij}(x_{ij},y_{ij})$ in the proposed program of equitable NDP can be obtained using the following formula:

$$t_{ij}(x_{ij},y_{ij}) = t_{ij}^0 \cdot \left(1 + \alpha\left(\frac{x_{ij}}{c_{ij}}\right)^{\beta}\right) = t_{ij}^0 \cdot \left(1 + \alpha\left(\frac{x_{ij}}{c_{ij}^0 + y_{ij}\cdot\Delta c_{ij}}\right)^{\beta}\right) \quad \forall (i,j) \in A \quad (19)$$

where α and β are parameters of the BPR function; c_{ij}^0 is the initial capacity of link (i,j); Δc_{ij} is a unit capacity expansion for existing or added Link (i,j).

Additionally, two remarks need to be made for the proposed equitable NDP model:

1) Two principal assumptions exist for the proposed model:

- For each link, travel speed remains the same at any point of the link.
- Network users have perfect knowledge of the traffic conditions over the network.

2) Mathematically, the upper-level program is an integer nonlinear programming problem and the lower-level one is a convex continuous optimization problem. The bi-level program, represented by Formulas (8)-(14), is a nonlinear, non-convex and intractable problem. Due to the intrinsic complexity of the bi-level model, previous solution algorithms have limitation in finding out globally optimal solutions. Thus, it is necessary to develop a dedicated exact solution methodology to solve the proposed bi-level model.

IV. Exact Solution Methodology

This section aims to develop an exact solution methodology for the developed bi-level equitable NDP model, which is able to find out the globally optimal solutions. This solution methodology comprises programming techniques including interior point method, branch and bound algorithm. It can be coded and implemented on the general-purpose optimization software AMPL with solvers IPOPT and COUENNE [28]. The major steps of the solution procedure as depicted in Fig. 2 are summarized into the following five steps:

Step 1 Initialization

Initialize iteration counter: $n = 0$;

Initialize decision variable set: $\boldsymbol{y} = \boldsymbol{y_n} = \boldsymbol{y_0}$.

Step 2 Obtaining flow patterns from the lower-level program

Solve the UE traffic assignment problem in the lower-level program based on the current NDP decision variable set $\boldsymbol{y_n}$, and obtain the flow pattern $\boldsymbol{x_n}$. The lower-level program, which is a convex continuous optimization problem, can be solved by interior point method. This method can be implemented by the solver IPOPT.

Step 3 Updating NDP decision variables in the upper-level program

Solve the transportation planning problem in the upper-level program based on the flow pattern $\boldsymbol{x_n}$ and get the updated NDP decision variable set $\boldsymbol{y_{n+1}}$. The upper-level program is an integer nonlinear programming problem. The solver COUENNE can be adopted to solve this problem through a branch and bound algorithm regardless of the convexity.

Step 4 Examining convergence

If the updated NDP decision variable set $\boldsymbol{y_{n+1}}$ is identical to the predecessor $\boldsymbol{y_n}$, then go to step 5. Otherwise, assign $\boldsymbol{y_{n+1}}$ to $\boldsymbol{y_n}$, reset the iteration counter n to be $n+1$ and return to *Step 2*.

Step 5 Outputting results

Output the NDP decision variable set $\boldsymbol{y_n}$ and corresponding flow pattern $\boldsymbol{x_n}$, as well as values of terms in the objective function (i.e. travel cost term $TSTT$, general equity term $SD(\tau)$ and network-scale-adjusted equity term $f(\boldsymbol{x},\boldsymbol{y})\cdot SD(\tau)$). Terminate the algorithm.

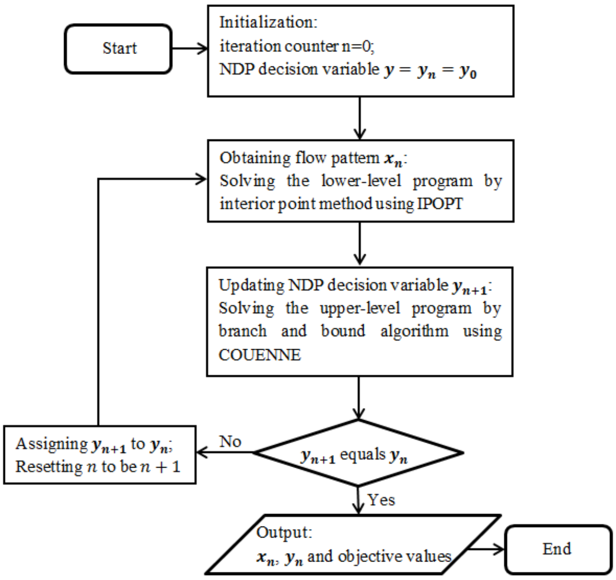

Figure 2. Flow chart of solution methodology

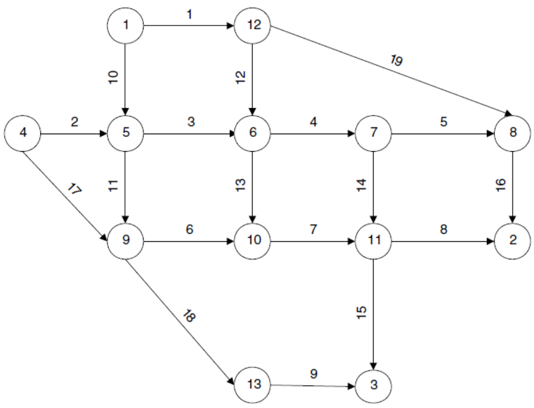

Figure 3. Topology of nguyen-dupuis network

In *Step 2* and *3*, interior point method and branch and bound algorithm, implemented using the solvers IPOPT and COUENNE, can achieve the globally optimal solutions to the lower-level and upper-level programs respectively. The loop of this algorithm continues until decision variable set $\boldsymbol{y}$ and its derived flow pattern $\boldsymbol{x}$ reach optima concurrently in the respective upper and lower levels. Therefore, the whole algorithm is an exact solution methodology, which can guarantee the global optimality of the final solutions. Moreover, the loop in this solution methodology reflects the interaction between the network planners and travellers in the decision process.

V. Case Analysis

In this section, a computational experiment is conducted on the Nguyen-Dupuis Network, in order to demonstrate the validity of the proposed approaches. The Nguyen-Dupuis Network is a middle-size network which has been used for extensive studies on NDP before. The topological graph of the network is depicted in Fig. 3 and parameters summarized in Table IV. Travel demands are listed in Table V.

TABLE IV. Link Parameters for Nguyen-Dupuis Network

Link ID	Length (mi)	Free Flow Speed (mi/h)	Free Flow Travel time (min)	Initial Capacity (veh/h)	Parameters in BPR function	
					α	**β**
1	4.08	35	7	2200	3.93	1
2	5.25	35	9	2200	2.44	1
3	5.25	35	9	2200	2.44	1
4	7	35	12	2200	0.92	1
5	1.75	35	3	2200	5.5	1
6	5.25	35	9	2200	1.83	1
7	2.91	35	5	2200	5.5	1
8	7.58	35	13	2200	0.85	1
9	2.92	35	5	2200	5.5	1
10	5.25	35	9	2200	3.06	1
11	5.25	35	9	2200	3.06	1
12	5.83	35	10	2200	1.1	1
13	5.25	35	9	2200	1.22	1
14	3.5	35	6	2200	0.92	1
15	5.25	35	9	2200	1.22	1
16	4.67	35	8	2200	2.75	1
17	4.08	35	7	2200	0.79	1
18	8.17	35	14	2200	1.57	1
19	6.42	35	11	2200	2	1

TABLE V. OD Demand

Origin \ Destination	2	3
1	1900	2100
4	2300	1700

In this example, binary variables with feasible region $\{0,1\}$ are adopted for all the decision variables y_{ij}. The value of y_{ij} means whether link (i,j) is improved by one-unit capacity expansion or not. Other parameters used in this NDP are summarized in Table VI.

TABLE VI. Parameters in NDP

Unit capacity expansion	$\Delta c_{ij} = 300,\ \forall (i,j) \in A$
Cost of unit capacity expansion	$\gamma_{ij} = 300,\ \forall (i,j) \in A$
Cost of planning policy	$\sum_{(i,j)\in A} \gamma_{ij}\cdot y_{ij} = \sum_{(i,j)\in A} 300\cdot y_{ij}$
Budget	$B = 1800$

The aim of this problem is to determine a planning policy in order to optimize the network above. To test the proposed bi-level program of equitable NDP and solution methodology, this problem is solved using approaches in this paper. The model and algorithm are coded in the optimization software AMPL with solvers IPOPT and COUENNE. The implementations have been made on Intel Core i7-4770 processor on Windows 7 platform. In this case, the average runtime spent in solving each NDP is less than 0.250 second. Computational results are summarized in Table VII.

TABLE VII. SOLUTION RESULTS OF EQUITABLE NDPS WITH DIFFERENT VALUES OF A

NDP Decision Variable	Modified Networks					Initial Network (Base Case)
	Weighting Factor α					
	1.0	0.8	0.5	0.2	0.0	
y_1	1	1	1	1	1	0
y_2	0	0	0	0	0	0
y_3	0	0	0	0	0	0
y_4	0	0	0	0	0	0
y_5	0	0	0	0	0	0
y_6	0	0	0	0	0	0
y_7	0	1	1	1	1	0
y_8	0	0	0	0	0	0
y_9	1	1	1	1	1	0
y_{10}	0	0	0	1	1	0
y_{11}	0	0	0	0	0	0
y_{12}	0	0	0	0	0	0
y_{13}	0	0	0	0	0	0
y_{14}	0	0	0	0	0	0
y_{15}	0	0	0	0	0	0
y_{16}	1	1	1	1	1	0
y_{17}	1	0	0	0	0	0
y_{18}	1	1	1	0	0	0
y_{19}	1	1	1	1	1	0
Travel Cost ($TSTT$)	716,793	717,012	717,012	719,452	719,452	753,816
General Equity ($SD(\tau)$)	2.20002	2.14217	2.14217	2.11725	2.11725	2.34959
Adjusted Equity ($f \cdot SD(\tau)$)	322949	314651	314651	311743	311743	351553
Runtime (s)	0.063	0.234	0.250	0.250	0.265	--

Table VII shows that different values of weighting factors α, equal to 0.0, 0.2, 0.5, 0.8, 1.0, are used to solve this equitable NDP. An important insight is that regardless of the values of weighting factors, all the planning policies can improve the system performance in terms of both travel cost and equity, with regards to that of the initial network. The percentage travel time savings as compared to the base case ranges from 4.6% to 4.9%. Meanwhile, the values of general equity term and adjusted equity term have been decreased by 6.4% ~ 9.9% and 8.1% ~ 11.3% respectively. Furthermore, it can be observed that adopting different weighting factors yields different link capacity expansions. This is because the weighting factor α is set to balance the importance of the two objectives, *i.e.* travel cost and equity. For the proposed bi-level program, the greater this weighting factor is, the more seriously the objective of travel cost is taken. It is shown in Table VII that as the weighting factor becomes higher, the total system travel time associated with the planning policy declines, and in other words, the traffic system performance in terms of travel costs ameliorates over the modified network.

From Table VII, it can be analysed that there is a significant trade-off between objectives of travel cost and equity. By accounting for the standard deviation of unit-length travel time, a benefit of improving the equity of the system performance is achieved, at the cost of increase in the overall travel cost. Moreover, the difference between travel costs associated with $\alpha = 1.0$ and 0.0 is 2,479 (719,452 minus 716,793), which is only a small proportion of decrease in travel cost in the modified network ($\alpha = 1.0$) as compared to the initial network, equal to 37,023 (753,816 minus 716,793). Thus, in order to make the design decisions on the network topology more equitable, the trade-off with respect to the slight increase in travel cost is acceptable.

In a word, the results show that the proposed approaches can generate solutions of higher quality in terms of travel cost and equity, with regards to the traffic performance over the initial network. Moreover, it demonstrates that the planning policies vary depending on the extent to which equity in the NDP is valued.

VI. Conclusion and Future Studies

In this paper, the network design problem accounting for equity has been studied. Equity associated with unit-length travel time is proposed and described mathematically, whose significance in the NDP is demonstrated over the Braess Network. A program for the equitable NDP problem is formulated into a nonlinear, non-convex bi-level programming model, where the planner's target is to minimize the weighted sum of the values of travel cost term and equity term. Then an exact solution methodology to the proposed model is developed based on programming techniques including interior point method, and branch and bound algorithm, which can guarantee the global optimality of the obtained solutions. Finally, the proposed approaches have been adopted to solve the NDP over the Nguyen-Dupuis Network. The result shows that the proposed approaches can yield desirable solutions with improvement in both travel cost and equity with regards to the initial network. It also demonstrates the significant differences made to the planning policies by considering equity in the NDPs. The model formulation and solution approach proposed in this paper provide systematic means for considering equity in NDPs and analysing explicit trade-offs between travel cost and equity, and as a result, technically support the government's transportation planning policies.

For future studies, the environment element could be integrated into the equitable NDP, in order to make the transportation system more sustainable. Homogeneous networks with regards to free flow travel time can also be studied in the context of equitable NDPs. Additionally, some economical measures, such as reversible lanes and congestion charges, could be considered in the process of network modification.

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