Parking Capacity Optimization Using Linear Programming

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***Abstract*—As the number of automobiles increases, the task of providing parking lots to accommodate more vehicles becomes very difficult and challenging. The design of parking lots needs more attention in order to have efficient and safe design. The capacity of a parking lot depends on many factors such as the parking angle. Choosing the most efficient parking angle is not an easy task since it varies with many factors such as the geometry and the circulation inside the parking lot. In this study, Integer Linear Programming (ILP)is used to determine the optimal parking angle. Different cases are formulated considering different layouts and dimensions in order to choose the parking angle which will provide the maximum number of spaces. Furthermore, the LINDO software is utilized to find out the optimal parking angle for each given area. The results showed thatLinear Programmingresulted in an enhanced the parking design since it allows for different combinations between the parking angles within the same parking facility. For future work, the formulation can be modified to suite irregular shapes within parking areas.**

***Index Terms*—parking design,linear programming,parking angle, parking capacity**

I. INTRODUCTION

Parking lotsare essential element of any transportation system as they have an impact on the overall performance of the transportation system. Accordingly, an efficient design of the parking lots will lead to a better performance of the whole transportation system. Nowadays, there is a significantly increasing demand for parking spacesdue to the increase in the number of automobiles. The shortage of the number of spaces created a challenge to utilize the parking areas more efficiently. Parking lots vary in design, size and location. The capacity of any parking lot depends on the number of vehicles it services, which is a function of the parking angle inside the parking lot.

Utilizing the parking lot space more efficiently results in higher capacity. Many factors affect the design a parking lot such as the number of vehicles that need to be accommodated, the parking angle, the circulation system within the parking lot, and the expected vehicle size. While developing any parking lot, the main objective is to provide the maximum capacity with a convenient and safe circulation. There are many problems associated with the parking design such as very high demand, shortage of land, and using the space inefficiently. The existing parking lots should be managed efficiently before building new parking facilities.

The parking stalls can be designed with different angles such as 45^0, 60^0, and 90^0, as shown in Fig. 1. The angle of the parking stalls has a great impact on the capacity and the circulation of the parking lot. The 60^0 parking angle is the most common angle because of the easy entry and exit of the parking stall. Also the 60^0 parking angle has a reasonable traffic lane width. The 90^0 parking angle provides a reasonable number of parking spaces for a given area. But due to the difficulty of entering and leaving the parking stall, it's only recommended for all-day parking such as university or work place parkingareas [1].

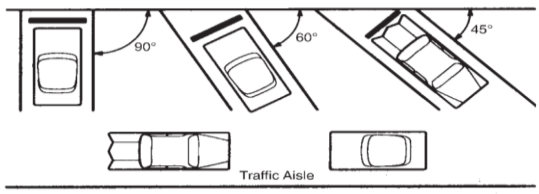

Figure 1. Parking lot angles [1].

Typical parking dimensions vary based on the angle of the parking. Each angle has a different stall length, width, and depth. Also the traffic lane for the parking varies for each parking angle.There are standard dimensions used for designing off-road parking lots. For example, the Dubai Municipality Manualrequires the minimum dimensions shown in Fig. 2 and Table I [2].

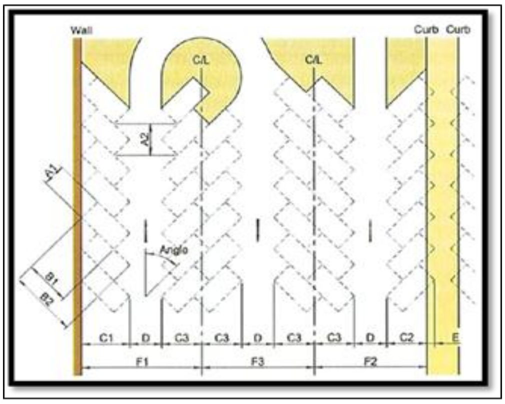

Figure 2. Angular parking layout/parameters [2].

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The one-way circulation system is preferred over the two-way circulation with angled parking. Although the two-way circulation provides more space for maneuvering, drivers may park in the space of the opposite direction therefore blocking circulations and creating conflicts. Drivers may violate the one-way aisle for the 90-degree parking because the maneuvering space needed for such parking is enough for two-waycirculation. Fig. 3 shows the suggested circulation configurations [3].

TABLE I. PARKING ANGLE DIMENSIONS [2].

Dimensions (for a bay size of 2.5m x 5.0m)	On Figure 8.4	Angle				
		30°	45°	60°	75°	90°
Bay width	A1	2.50	2.50	2.50	2.50	2.50
Bay width, parallel to aisle	A2	5.60	3.50	2.80	2.70	2.50
Bay length	B1	5.00	5.00	5.00	5.00	5.00
Length of line between bays	B2	10.00	7.50	6.25	5.65	5.00
Bay depth to wall	C1	4.50	5.30	5.60	5.50	5.00
Bay depth to curb	C2	4.15	4.70	4.90	4.75	4.25
Bay depth to interlock	C3	3.40	4.40	5.05	5.15	5.00
Aisle width between bay lines	D	3.50	3.75	4.50	6.00	7.00
Bumper overhang (typical)	E	0.35	0.60	0.70	0.75	0.75
Module, wall to interlock	F1	11.40	13.45	15.65	17.75	18.00
Module, curb to interlock	F2	11.05	12.85	14.95	17.00	17.25
Module, interlock to interlock	F3	10.30	12.55	14.80	17.40	18.00

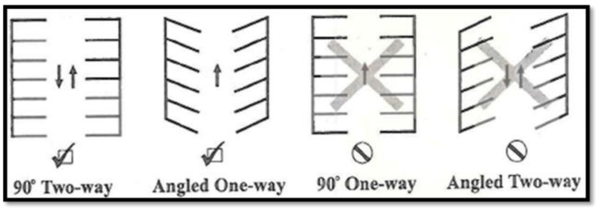

Figure 3. Suggested circulation configurations [3].

II. LITERATURE REVIEW

Many studies have investigated the impact of the parking angle on the overall efficiency of a parking lot. In other words, which parking angle will provide the maximum number of spaces. One study was carried out by the Department of Transportation in the City of Los Angeles to find the optimal parking angle for a given lot dimension. The angle dimensions used in this study were based on the City of Los Angeles's Department of Building and Safety standards. In this study, many calculations were performed on parking lots with different dimensions in order the find the most efficient parking angle in terms of number of parking spaces. By the assistance of a personal computer, forty rectangular parking areas were evaluated in the study. The length of the parking lots varies from 100 to 500 feet with an increment of 25 and 50 feet. And the width of the parking lots varies from 100 to 200 feet with an increment of 25 feet. The 90-degree parking angle resulted in the maximum number of spaces in most of the cases. In addition, the 75-degree angle was the second most efficient parking angle. Table II summarizes the parking spaces efficiency based on each parking angle. The study concluded that in 67% of the cases, the 90-degree parking angle generated the maximum number of spaces. The outcome of this study was based on arranging the parking aisles parallel to the shorter dimension [4].

TABLE II. SUMMARY OF PARKING SPACE EFFICIENCY [4].

Parking Angle (degrees)	*Maximum Parking Produced (%)*
90	67
75	23
60	8
45	2
30	0

Furthermore, anotherinvestigated the optimal parking angle using mathematical analysis field tests. In the mathematical model, the area required for each stall was formulated as a function of the parking angle.By differentiation, the angle which will minimize that area is the optimal angle and it turned out to be 70-degree parking angle. In addition to that, in terms of maneuverability and safety, the 70-degree parking angle is more preferable over the 90-degree parking angle. [5]

In addition to that, a research effort introduced a new approach for obtaining the maximum capacity of a corner parking lot surrounded by two streets. In order to create the model, a set of nonlinear equations was developed with certain reasonable assumptions. The equations can handle different combinations of car sizes, and different stall angles. A factor of measuring the easiness of parking maneuvers was introduced to select to safest layout if several layouts give the same maximum capacity [6].

Another research introduced the idea of having automatic planning and manual adjustment to find the optimal design of parking lots. The automatic planning aims to get the maximum number of spaces by dividing the lot into appropriate number of rectangles and plan them separately. The manual planning is mainly to remove the parking spaces which are occupied by any obstacles or blocking passages after combining all the sub rectangles together. Also, the effectiveness and the feasibility of the method were verified through a case study [7].

Linear Programmingis an effective tool to optimize different problems. A case study considered the optimization of the available parking area by using Linear Programming. The average parking accumulation and the average parking duration were obtained from the field survey in order to develop the model. The objective function is to maximize parking stall capacity to be allocatedfor each type of vehicle within the available parking space area. Three scenarios were tested to determine the optimal solution for each scenario. The scenarios are considering parking accumulation only, considering parking duration only, and considering both parking accumulation and parking duration. Each scenario produced different results since each formulation considered various aspects [8].

Another research effort discussed the optimization of parking lots considering parking behavior. The paper analyzed many factors such as walking distance from the parking, parking security, parking charge, etc. based on the result of the main factors impacting parking choice, a parking choice probability formula was developed. Furthermore, parking survey data has been used to calibrate unknown parameters of the optimization model. The utilization rate of the parking lot has increased by 71.4% on weekdays and 66.7% on weekdays. The optimization model can evidently improve the utilization rate of parking lots [9].

III. Problem Statement

Designing the parking lots with the optimal parking angle is extremely important because it helps inutilizing the space more efficiently. Increasing the number of parking stalls for the existing parking lots before building a new parking facility will reduce the land consumption as it is more economically efficient. Furthermore, parking users will be better off because having an efficient parking design will reduce the time looking for a parking space. Also, if more parking stalls are provided within the same parking area, this will help to reduce air and noise pollution. Finally, parking violations within the parking lot will be reduced dramatically.

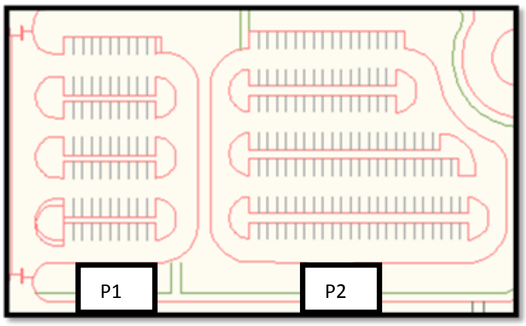

Figure 4. P1 & P2 parking layout at AUS (existing design).

In fact, using an optimal parking angle has a huge impact on adding more parking spaces for the same parking area. For example, Parking area 1 (P1)and Parking area 2 (P2) are two paid parking lots at the American University of Sharjah (AUS). Now, both parking lots are separated by an area and currently they have a 90-degree parking angle and 2-way circulation parking system, as depicted in Fig. 4. A new design was proposed tocombine the two parking lots together. Also, the proposed design suggests a 1-way circulation across the aisles and 60-degree parking angle Fig. 5. The new design was selected based on different designs using different parking angles. The new design with 60-degree parking angle increased the total number of parking spaces form 263 to 358, which corresponds to about 36% increase in the parking capacity.

This case study investigates the utilization Linear Programming techniques in order to improve the parking capacity beyond the level achieved by the new design. The study investigates three different cases to choose the optimal parking angle. Each case has different configuration of the parking area such as the circulation between the aisles and the location of the entrance and the exit. Five parking angles are evaluated to determine which combination will provide the maximum number of spaces.

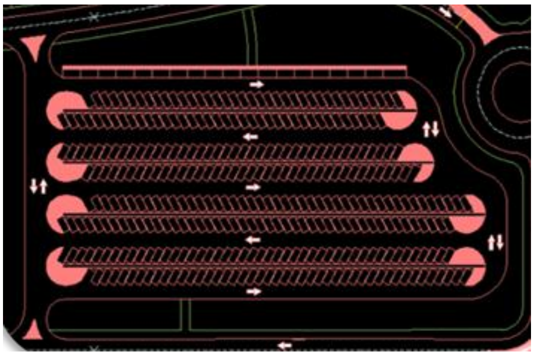

Figure 5. P1 & P2 parking layout at AUS (proposed design).

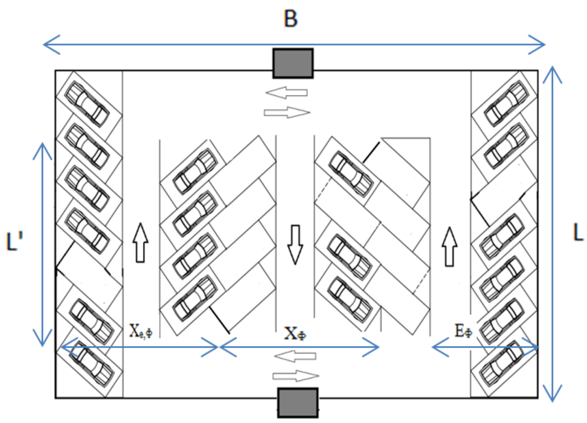

Figure 6. Parking layout for Case 1.

IV. Methodology

Integer Linear Programming (ILP) will be used for the three cases. In case 1 and 2, the three decision variables are the number of full interior rows, the number of full exterior rows, and the number of exterior rows. While in case 3 the decision variables are the number of full interior rows and the number of exterior rows only. The objective function is to maximize the total number of parking stalls for each case. The constraints in this study are the width and the length of the parking lot.Also, the total number of the exterior rows can't be greater than 2. Three different parking dimensions will be evaluated for each case (i.e.120x110 m, 120x80 m, and 90x60 m). Alsothe LINDO software will be used in order to find the maximum parking capacity and the corresponding parking angle for each given area.

All the decision variablesare restricted to be integer numbers, because it's not practical to have the number of rows or stalls as a fraction. Using the Integer Linear Programingensures that each row has the same parking angle. Mixing angles within the same row is not practical. On the other hand, allowing different parking angles within the same parking facility should be considered.Having different angles within the parking facility might improve the design and make it more efficient.

Furthermore, the traditional approach for the parking design was followed. AutoCAD drawings were developed for each parking dimension and the five parking angles were evaluated (i.e. 30^o, 45^0, 60^o, 75^0 and 90^o) to compare which angle will provide the maximum number of parking stalls. Finally, the LINDO output was compared with the AutoCAD results to find out which method will provide the maximum number of spaces.

A. *Case 1:-*

- 1-way circulation across the aisles
- 2-way circulation perpendicular to the rows.
- Two entries/exits located at the both sides of the parking lot.
- The exterior rowsare adjacent to the parking boundary.

Fig. 6 shows a schematic representation of Case 1.

Decision variables:

X_Φ= number of full interior rows with Φ^o angle.
$X_{e,\Phi}$= number of full exterior rows with Φ^o angle.
E_Φ= number of exterior rows with Φ^o angle.
n_Φ= number of full interior bays with Φ^o angle.
$n_{e,\Phi}$= number of full exterior bays with Φ^o angle.
$n_{EE,\Phi}$= number of exterior bays with Φ^o angle.

$\Phi = 30^0, 450, 60^0, 75^0$ or 90^o

Objective function:

Max. $z=\Sigma(n_\Phi + n_{e,\Phi} + n_{EE,\Phi})$

Constraints variables:

$10.3X_{30}+12.55X_{45}+14.8X_{60}+17.4X_{75}+18X_{90}+11.4X_{e,30}+13.45X_{e,45}+15.65X_{e,60}+17.75X_{e,75}+18X_{e,90}+8E_{30}+9.05E_{45}+10.1E_{60}+11.5E_{75}+12E_{90}\leq B$
$5.6\ n_{30} - 2X_{30}\ L' \leq 0$
$3.5n_{45}-2X_{45}L' \leq 0$
$2.8\ n_{60} - 2X_{60}\ L' \leq 0$
$2.7n_{75}-2X_{75}L' \leq 0$
$2.5\ n_{90} - 2X_{90}\ L' \leq 0$
$5.6\ n_{e,30}- X_{e,30}\ L - X_{e,30}\ L' \leq 0$
$3.5n_{e,45}-X_{e,45}L -X_{e,45}L' \leq 0$
$2.8\ n_{e,60}- X_{e,60}\ L - X_{e,60}L' \leq 0$
$2.7\ n_{e,75}- X_{e,75}\ L - X_{e,75}\ L' \leq 0$
$2.5\ n_{e,90}- X_{e,90}\ L - X_{e,90}\ L' \leq 0$
$5.6\ n_{EE,30}- E_{30}\ L \leq 0$
$3.5\ n_{EE,45}- E_{45}\ L \leq 0$
$2.8\ n_{EE,60}- E_{60}\ L \leq 0$
$2.7\ n_{EE,75}- E_{75}\ L \leq 0$
$2.5\ n_{EE,90}- E_{90}\ L \leq 0$
$Xe_{,30}+X_{e,45}+X_{e,60}+X_{e,75}+X_{e,90}+E_{30}+E_{45}+E_{60}+E_{75}+E_{90}\leq 2$

B. *Case 2:-*

- 1-way circulation across the aisles
- 1-way circulation perpendicular to the rows.
- Two entries/exits located at the both sides of the parking lot.
- The exterior rows are adjacent to the wall.

Decision variables:

X_Φ= number of full interior rows with Φ^o angle.
$X_{e,\Phi}$= number of full exterior rows with Φ^o angle.
E_Φ= number of exterior rows with Φ^o angle.
n_Φ= number of full interior bays with Φ^o angle.
$n_{e,\Phi}$= number of full exterior bays with Φ^o angle.
$n_{EE,\Phi}$= number of exterior bays with Φ^o angle.

$\Phi = 30^0, 450, 60^0, 75^0$ or 90^o

Objective function:

Max. $z= \Sigma\ (n_\Phi + n_{e,\Phi} + n_{EE,\Phi})$

Constraints variables:

$10.3X_{30}+12.55X_{45}+14.8X_{60}+17.4X_{75}+18X_{90}+11.4X_{e,30}+13.45X_{e,45}+15.65X_{e,60}+17.75X_{e,75}+18X_{e,90}+8E_{30}+9.05E_{45}+10.1E_{60}+11.5E_{75}+12E_{90}\leq B$
$5.6\ n_{30} - 2X_{30}\ L' \leq 0$
$3.5n_{45}-2X_{45}L' \leq 0$
$2.8\ n_{60} - 2X_{60}\ L' \leq 0$
$2.7n_{75}-2X_{75}L' \leq 0$
$2.5\ n_{90} - 2X_{90}\ L' \leq 0$
$5.6\ n_{e,30}- X_{e,30}\ L - X_{e,30}\ L' \leq 0$
$3.5n_{e,45}-X_{e,45}\ L -X_{e,45}\ L' \leq 0$
$2.8\ n_{e,60}- X_{e,60}\ L - X_{e,60}L' \leq 0$
$2.7\ n_{e,75}- X_{e,75}\ L - X_{e,75}\ L' \leq 0$
$2.5\ n_{e,90}- X_{e,90}\ L - X_{e,90}\ L' \leq 0$
$5.6\ n_{EE,30}- E_{30}\ L \leq 0$
$3.5\ n_{EE,45}- E_{45}\ L \leq 0$
$2.8\ n_{EE,60}- E_{60}\ L \leq 0$
$2.7\ n_{EE,75}- E_{75}\ L \leq 0$
$2.5\ n_{EE,90}- E_{90}\ L \leq 0$
$Xe_{,30}+X_{e,45}+X_{e,60}+X_{e,75}+X_{e,90}+E_{30}+E_{45}+E_{60}+E_{75}+E_{90}\leq 2$
$X_{30}+X_{45}+X_{60}+X_{75}+X_{90} -3Y=0$

C. *Case 3:-*

- 1-way circulation across the aisles
- 2-way circulation perpendicular to the rows.
- Two entries/exits located at the both sides of the parking lot.
- The exterior rows are not adjacent to the wall.

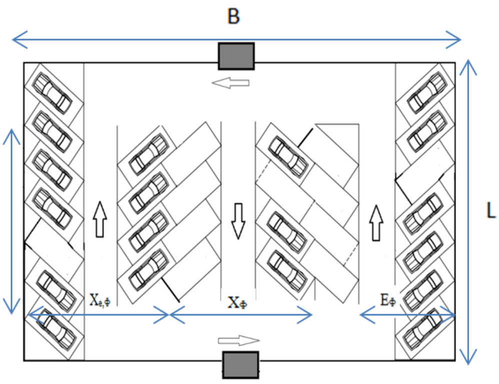

Figure 7. Parking layout for Case 2.

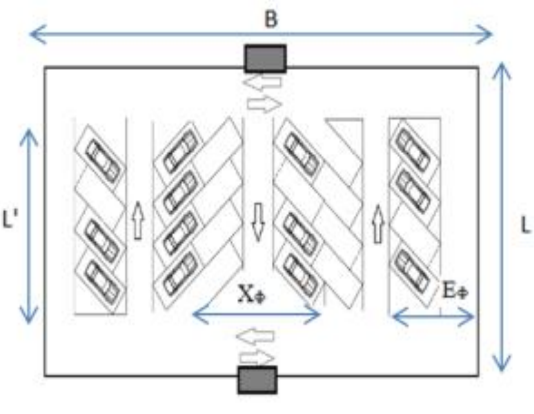

Figure 8. Parking layout for Case 3.

Decision variables:-

X_{Φ}= number of full interior rows with Φ^{o} angle.

E_{Φ}= number of exterior rows with Φ^{o} angle.

n_{Φ}= number of full interior bays with Φ^{o} angle.

$n_{EE,\Phi}$= number of exterior bays with Φ^{o} angle.

$\Phi = 30^{0}$, 450, 60^{0}, 75^{0} or 90^{o}

Objective function:-

Max. $z= \Sigma\ (n_{\Phi}+ n_{EE,\Phi})$

Constraints variables:-

$10.3X_{30}+12.55X_{45}+14.8X_{60}+17.4X_{75}+18X_{90}+8E_{30}+9.05E_{45}+10.1E_{60}+11.5E_{75}+12E_{90}\leq B$

$5.6\ n_{30} - 2X_{30}\ L' \leq 0$

$3.5n_{45}-2X_{45}L' \leq 0$

$2.8\ n_{60} - 2X_{60}\ L' \leq 0$

$2.7n_{75}-2X_{75}L' \leq 0$

$2.5\ n_{90} - 2X_{90}\ L' \leq 0$

$5.6\ n_{EE,30}- E_{30}\ L' \leq 0$

$3.5\ n_{EE,45}- E_{45}\ L' \leq 0$

$2.8\ n_{EE,60}- E_{60}\ L' \leq 0$

$2.7\ n_{EE,75}- E_{75}\ L' \leq 0$

$2.5\ n_{EE,90}- E_{90}\ L' \leq 0$

$E_{30}+E_{45}+E_{60}+E_{75}+E_{90}\leq 2$

D. Tradional Approach

The AutoCAD has been used in order to determine the maximum number of parking spaces that can be provided for each angle. Three different parking dimensions were investigated in this case study (i.e. 120x110 m, 120x80 m, 90x60 m). As shown in Table 3, the 90-degree and the 60-degree parking angles provided the same number of parking spaces for the area of 120x110m. While for the parking dimension of 120x80m, the 90-degree parking angle provided more spaces. Finally, the 60-degree parking angle provided the maximum number of parking spaces for the dimension of 90x60m.

TABLE III: AUTOCAD RESULTS FOR EACH ANGLE.

	30°	45°	60°	75°	90°
120x110	399	486	504	377	504
120x80	294	342	360	260	378
90x60	150	182	208	140	180

V. RESULTS AND DISCUSSION

In order to maximize the total number of parking spaces, LINDO software has been used to determine the number of parking stalls corresponding to each angle. In each case, the number of rows was determined for each angle and the corresponding number of spaces.The number of rows and bays are restricted to be integer. Mixing angles within the same row is not permissible because each row is restricted only to one parking angle. In case of having different design angles (i.e. 60 and 90 degrees) within the same parking facility, engineering judgment and experience will be used to arrange the rows. It's recommended to arrange the rows with the same parking angle next to each other to avoid confusion for parking users.

TABLE IV: SUMMARY OF THE LINDO AND AUTOCAD OUTPUTS.

	Case 1			Case 2			Case 3			AutoCAD		
	120x110	120x80	90x60	120x110	120x80	90x60	120x110	120x80	90x60	120x110	120x80	90x60
Total number of parking stalls	**558**	**397**	**272**	**552**	**397**	**272**	**547**	**393**	**217**	**504**	**378**	**208**
N30	0	0	0	0	0	0	0	0	0	0	0	0
N45	0	0	43	0	0	43	0	242	0	0	0	0
N60	378	227	108	378	227	108	378	151	217	0	0	104
N75	0	0	0	0	0	0	0	0	0	0	0	0
N90	0	0	0	84	0	0	169	0	0	336	252	0
NE30	0	0	0	0	0	0	-	-	-	0	0	0
NE45	0	0	0	0	0	0	-	-	-	0	0	0
NE60	0	80	0	0	80	0	-	-	-	0	0	104
NE75	0	0	0	0	0	0	-	-	-	0	0	0
NE90	180	90	0	90	90	0	-	-	-	168	84	0
NEE30	0	0	121	0	0	121	0	0	0	0	0	0
NEE45	0	0	0	0	0	0	0	0	0	0	0	0
NEE60	0	0	0	0	0	0	0	0	0	0	0	0
NEE75	0	0	0	0	0	0	0	0	0	0	0	0
NEE90	0	0	0	0	0	0	0	0	0	0	42	0
X30	0	0	0	0	0	0	0	0	0	0	0	0
X45	0	0	1	0	0	1	0	4	0	0	0	0
X60	5	3	2	5	3	2	5	2	4	0	0	2
X75	0	0	0	0	0	0	0	0	0	0	0	0
X90	0	0	0	1	0	0	2	0	0	4	3	0
XE30	0	0	0	0	0	0	-	-	-	0	0	0
XE45	0	0	0	0	0	0	-	-	-	0	0	0
XE60	0	1	0	0	1	0	-	-	-	0	0	2
XE75	0	0	0	0	0	0	-	-	-	0	0	0
XE90	2	1	0	1	1	0	-	-	-	2	1	0
E30	0	0	2	0	0	2	0	0	0	0	0	0
E45	0	0	0	0	0	0	0	0	0	0	0	0
E60	0	0	0	0	0	0	0	0	0	0	0	0
E75	0	0	0	0	0	0	0	0	0	0	0	0
E90	0	0	0	0	0	0	0	0	0	0	1	0

As shown in the Table IV, Integer Linear Programming providedmore parking stalls compared to the traditional approach results.Using combinations between parking angles enhanced the design of the parking lot. Most of the stalls are with 60-degree parking angle since it has a reasonable traffic lane width.

As shown in Table IV, all of the three cases provided almost the same number of stalls for the dimensions of 120x110 m and 120x80 m. While with the dimension of 90x60 m, case 1and case 2 provided more parking spaces compared to case 3.

VI. Conclusion

In this case study, Integer Linear Programming has been used to find the optimal parking angles for a parking lot with a given dimensions. Three different cases were formulated with different parking layouts. For each case, 30˚, 45˚, 60˚,75˚ and 90-degree parking angles were evaluated to determine the number of stalls for each parking angle. The constraints for each case were determined based on the dimensions of the parking area.

The LINDO output was compared with the AutoCAD drawings for each parking dimensions. The Linear Programming provided more parking spaces because it allows a combination between the parking angles.

VII. Recommendations

- More cases can be formulated by changing the circulation inside the parking lot or changing the location of the entrance or the exit.
- For large parking area, the formulation can be modified to limit the length of each row to a certain value; otherwise, a U-turn can be introduced.
- The formulation can be modified to suit a multistory parking where there are restrictions on the area because of the existence of the columns.
- The formulation can be modified to suit irregular shapes, which may require a non-linear formulation to be considered for such cases.

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