Optimal Route Selection Model Based on Multiple Criteria Approach

Alireza Naseri
Department of Civil Engineering, Tabriz Branch, Islamic Azad University, Tabriz, Iran
Email: alinaseri@iaut.ac.ir

Afsaneh Mokhtari
M. A in Architecture, Abhar Branch, Islamic Azad University, Abhar, Iran
Email: naseri@gmx.com

***Abstract*—Network is an appropriate tool for making decisions on diverse and complex problems. Analysis of these problems through other methods is difficult and it sometimes needs complicated calculations. There are many methods to find the best route on a network. The methods have been designed based on a criterion, for instance, distance, cost, time, etc, as values of each criterion is additive. However, if a decision-maker intends to consider several criteria, which are sometimes conflicting and some of them are non-additive, how will route selection method be? The present article attempted to present a model to find optimal route for these types of networks.**

***Index Terms*—multi criteria decision making, network, optimal route**

I. Introduction

Network models and analyses are extensively used in operation research. Analysis and design of large-scale irrigation systems, transportation networks, computer networks, flight networks, and terrestrial and satellite networks are some applications of networks. Network performance methodology is used for solving industrial issues such as warehousing and goods distribution, project planning, replacing equipment, cost control, traffic study, queue analysis (queuing), assembly line, and human resource allocation [1]. On the ever-increasing use of network techniques, Pritsker, states that networks and network analyses play a crucial and developing role in describing and improving preliminary operational systems, as a real system can be modeled using a network. The following items may cause such ever-increasing application of networks:

- Complex systems can be modeled by combining simple systems.
- Need for communication mechanism to discuss on an operational system in terms of the important features of a network.
- A tool to specify required data for system analysis.
- A starting point for analyzing and scheduling an operational system [2].

Philips & Diaz, explain impressiveness of using network models as follows:

- Network models introduce many real systems accurately.
- Most unskilled people accept network models more conveniently than other models of operation research. Moreover, as network models are often related to physical issues, they are explained easily to those who take advantage of little quantitative background.
- Network algorithms facilitate extremely efficient solutions to solve some large-scale models.

In most cases, network models can solve some problems, which have many variables and limitations, whereas solving them using other optimization techniques would be very complicated. This is so because network approach, in most cases, provides an opportunity to design a customized structure for a problem [3].

II. Problem Definition

In case there is only one criterion for the best route in a network, that problem is called shortest route problem. The relevant criterion may be distance, time, cost, or any other criterion aiming to minimize sets of branches on a route. Now, if there are two or many criteria and possibly conflicting ones instead of a single criterion for choosing a route with different measurement criteria, while values of one or many criteria are non-additive, how can we choose the best route? Obviously, it is not possible to use the term «shortest» because there are more than one criterion in this condition. Here, one criterion may prefer maximization while the other may favor minimization. In this condition, the term «the best route» is used instead of «the shortest route». For example, we intend to move from a source node to a destination node in a network and three different criteria including distance, cost, and safety are considered. The first two criteria (distance and cost) are negative and quantitative. The third criterion (safety) is positive and qualitative. Now, how can we specify the best route with respect to these three criteria and taking weight (important factor) into account?

This is of MCDM problems because different criteria are considered for making decision on the best route.

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Meanwhile, it is a special type of MCDM problems, as each branch of the network is placed on the route. In other words, it takes advantage of a zero and one logic [4].

III. Background

There are different methods, such as linear programming and Dikestra's algorithm, to solve the shortest route problems in a network based on a criterion (objective). In linear programming method, each variable includes selection or lack of different branches. An objective function includes total number of coefficients of branches (distance, cost, time, etc) multiplied by the variable related to selecting or deselecting a branch. In linear programming, there are some limitations for each branch. If a variable is placed on the left, values 1, 0, and (-1) for source node, middle nodes, target node are placed on the right.

Dikestra's algorithm (marking method) for solving the shortest route problems is according to a criterion that is based on two types of markings: temporary and permanent. Temporary mark indicates that a route is specified from a source to a certain node and permanent mark shows how far the shortest distance between a source and a certain node is. Dikestra's algorithm can be used only if the coefficients of network branches are non-negative [5]. If there are negative coefficients for all or some branches in a network, Dikestra's algorithm cannot be used because by moving from the source node to destination, distance may be shortened and consequently we reach the destination node without thinking over other routes. In these types of networks (appearance of branch with negative length), «dual» algorithm is used for solving problems [6].

Linear programming is the basis for this algorithm. Problem solving starts with a super-optimal solution. As it does not satisfy limitations of dual problem, all the branches of the dual algorithm are taken into consideration. Such procedure is iterated as long as the shortest distance and routes are found. In case there is generally more than an objective in networks, a multi-criteria utility function can be used for solving this problem. Then it is solved as one objective. Here, all the effective solutions can be generated for the assumed problem and may be introduced to the decision-maker to select the most appropriate route [7]. In 1982, Klingman and Moot presented a subprogram for basic solution to solve network problem test as being effective or non-effective. It can be used for every adjacent basic solution [8]. In 1978, Diaz proposed the use of compromise solutions to solve multi-objective network problems. A compromise solution compares closeness of a solution to the ideal solution (optimization of each target function separately) using a solution [9].

IV. Method

None of the methods mentioned earlier can be used in their preliminary form for finding route based on the non-additive criteria. Following points should be considered to select the best route based on different and probably conflicting criteria in a network:

- Selecting a branch in a network based on zero and one logic (selection or non-selection of a branch.
- Values of branches of a route may be additive as far as a criterion is concerned; however, they might not be additive in terms of other criterion [10]. For example, assume two distance criteria (in terms of kilometer) and safety (in terms of probability). One route is additive in terms of distance, it means that the distances of route branches can be added and obtain the route distance; however, it is not the case for safety. See Fig. 1.

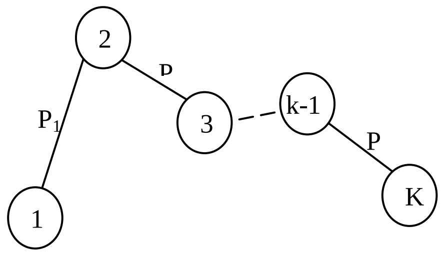

Figure 1. Safety criterion

If reliabilities (safety) of 1-2, 2-3, and (K-1)-K branches are shown by P_1, and P_2, and P_k, respectively, the overall reliability of the route from group 1 to group k, which is shown by P_{lk} will be as follows:

$$P_{lk} = P_1 \times P_2 \times ... \times P_k . \quad (1)$$

In this case, such problem can be resolved using a logarithmic transformation, which changes product of probabilities into addition. Such transformation will be as follows:

$$\log P_{1k} = \log P_1 + \log P_2 + ... + \log P_k . \quad (2)$$

Mathematically, max P_{lk} equals Max log P_{lk} and as safety is expressed in terms of probability, P_{lk} is a value between zero and one. As log $P_{lk}<0$, Max log P_{lk} equals Min (-log P_{lk}). Therefore, using this transformation, P_i probability in the network equals Min (-log P_i). Consequently, P_i probability in the network is replaced by (-log P_i) [11].

In case a non-additive criterion, such as safety is expressed by terms such as very low, low, average, etc, it will be necessary to transform them into probability numbers (between zero and one) using the following scale.

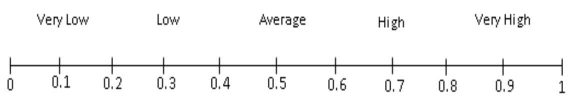

Figure 2. Quality scale grading in network.

In case there is a criterion with negative aspect in a network, it is necessary to transform it into qualitative index with positive aspect. Complementation method can be used for this purpose. For instance, difficulty with 0.3 equals convenience with 0.7. (1-0.3=0.7)

With respect to the above arrangements, following algorithm can be used to access the best route in multi-criteria networks.

Step 1: Creating matrix of values D= [r_{ij}] in which D is decision matrix and r_{ij} is the value caused by selecting i branch with respect to j. Following items should be observed respectively in creating the matrix:

- All the values of qualitative indices should be expressed in numbers.
- Values of qualitative criterion with negative aspects should be transformed into values with positive aspect through complementation method.
- (-log) of qualitative criteria values should be calculated and replaced.

Step 2: Scale-up of the values related to each criterion and creating D_n [n_{ij}] where :

$$n_{ij} = \frac{r_{ni}}{\max r_{ij}} \tag{3}$$

Step 3: Calculation of weighted mean of values of I branch using indices weight diagram is as:

$$\mu_i = \sum W_i \times n_{ij} \tag{4}$$

where W_i is weight criterion of j.

Step 4: finding the shortest route in a network using linear programming method and/or marking.

Above algorithm makes additive values of different branches. (It is performed in step 1). Through the following two steps (2 and 3), the values are scale-up and the scale-up values for each branch are combined with respect to their weight. Finally, step 4 provides conditions to find the best route through one of the classical methods, for instance linear programming or marking.

V. Example

Diagram No 2 shows a network that aims to find the best route with respect to four criteria of cost, distance, safety, and difficulty. Important factor of different indices, Wj, is as follows:

Wj= (0.3, 0.1, 0.4, and 0.2)

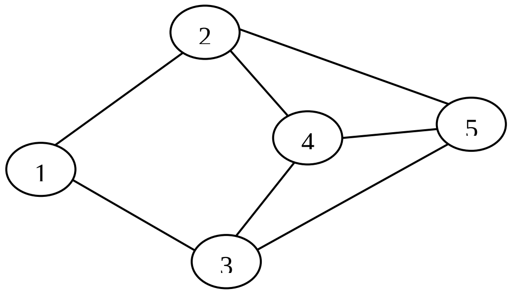

Figure 3. Example network

Table I exhibits the information related to each branch in terms of four different criteria.

TABLE I. Information on Network

Branch	Cost x_1^-	Distance (km) x_2^-	Safety x_3^+	Difficulty x_4^-
1-2	50	10	High	Average
1-3	50	12	High	Low
2-4	20	4	Average	High
2-5	20	15	Average to High	Very Low
3-4	40	6	Very High	Low
3-5	20	8	Average	Average
4-5	30	5	Average to Very High	Very low

TABLE II. Operation Of Step 1

Branch	Safety x_3^+	$(-\log x_3^{-})^{-}$	Difficulty x_4^-	Convenience x_4^+	$(-\log x_4)^{-}$
1-2	0.7	0.155	0.5	0.5	0.301
1-3	0.7	0.155	0.3	0.7	0.155
2-4	0.5	0.301	0.7	0.3	0.523
2-5	0.6	0.222	0.1	0.9	0.046
3-4	0.9	0.046	0.3	0.7	0.155
3-5	0.5	0.301	0.5	0.5	0.301
4-5	0.8	0.097	0.1	0.9	0.046

Now**,** *we solve the example using the algorithm.*

Step 1: Safety and difficulty scales are qualitative. Therefore, following measures are required:

- Values of these criteria should be transformed into quantitative values using the range introduced earlier for the qualitative indices.
- For qualitative criterion of difficulty that has negative aspect (the more difficult it is, the more unfavorable it will be.), it is changed into qualitative index or positive aspect using complementation method.

- (-log) is calculated and replaced for two qualitative indices of safety and difficulty (transformed easily by complementation). Table II shows the operation.

Therefore, values matrix for the four criteria, which all have negative aspects now, can be formed. Table III shows the values matrix.

TABLE III. MATRIX OF VALUES

Branch	x_1^-	x_2^-	x_3^-	x_4^-
1-2	50	10	0.155	0.301
1-3	50	12	0.155	0.155
2-4	20	4	0.301	0.523
2-5	90	15	0.220	0.046
3-4	40	6	0.046	0.155
3-5	20	8	0.301	0.310
4-5	30	5	0.097	0.046

Steps 2: Now, the values related to each criterion are scale-up using the following formula: For example for distance index (X_1)

n_{1-1}, , n_{1-2} and , … ,and

Step 3: Values related to each branch are combined with respect to weights of indices W= (0.3, 0.1, 0.4, and 0.2) using the following formula:

For example, for 1-2 branches, we have

µ1-2= (0.3) (0.556) + 0.1(0.667) + 0.4 (0.5 15) + 02(0.576) =0.5547

Table IV shows the results of step 2 and 4.

TABLE IV. RESULTS OF STEPS 3 AND 4

Branch	x_1^-	x_2^-	x_3^-	x_4^-	μ_i
1-2	0.556	0.667	0.515	0.576	0.5547
1-3	0.556	0.800	0.515	0.296	0.5120
2-4	0.220	0.267	1	1	0.6933
2-5	1	1	0.738	0.088	0.7128
3-4	0.444	0.400	0.153	0.296	0.2936
3-5	0.222	0.533	1	0.576	0.6351
4-5	0.333	0.333	0.322	0.088	0.2796

Step 4: The shortest route is obtained using marking method:

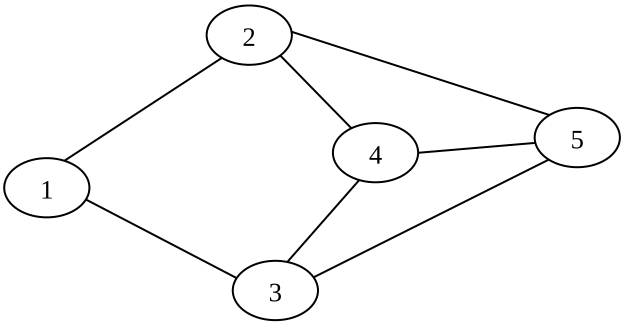

Figure 4. Finding the shortest route

Therefore, with respect to the four criteria, 1-3-4-5 route is the best route in the network.

VI. CONCLUSION

If there are different (quantitative, qualitative, positive, and negative) criteria to find the best route in a network, it is possible to score qualitative indices between 0 and 1 using the scale introduced earlier. Qualitative scales are not usually additive in networks; therefore, it is necessary to make them additive using logarithmic transformation. In case the qualitative scale has negative aspect, it is possible to convert it into a positive aspect after scoring using complementation method and then use logarithmic transformation. If the values of all network criteria are additive and they are transformed into the criteria of negative aspects, it is possible to scale-up every branch and then combine them using weight of criteria so that a value is obtained for each branch. Afterwards, the shortest route classical methods, for example, linear programming or marking method, can be applied to find the best route.

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Alireza Naseri was born in Tehran, Iran in 1980 and received his B.Sc. and M.S. degrees in Civil Engineering from Islamic Azad University, Iran in 2002 and 2006, respectively. He then continues his graduate study in Civil Engineering at Tabriz University, Iran. Naseri is with Dept. of Civil Eng, Islamic Azad University, Tabriz Branch as an Academic Staff Member from 2008 to present. His research interests in Transportation include Intelligent Transportation Systems, Traffic Modeling, Sustainable Transport and Traffic Network Management. His researches outcomes have been published over 20 articles, 1 book and 1 International Award for Invention.

Naseri is a member of Iranian Society of Transportation and Traffic Engineering. Also, he is a member of Iranian Association of Rail Transport Engineering, Iranian Society of Transportation Engineers and International Federation of Inventors Associations.

