Vehicle Routing Problem with Heterogeneous Customers Demand and External Transportation Costs

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***Abstract*—The paper presents the problem of transportation service optimization with heterogeneous customers demand and external transportation costs. Main attention is paid to the external transport costs generated by road transport and to the problem of certain goods groups transport on the same vehicle. Optimization task with the criteria of transportation costs minimization was formulated. To solve the problem authors used the clustering procedure and genetic algorithm. Computational results were obtained with the usage of authors' computer application.**

***Index Terms*—vehicle routing problem, genetic algorithm, clustering, external transportation costs, heterogeneous demand**

I. Introduction

The Vehicle Routing Problem (VRP) is an extension of the one of the oldest optimization problems called The Traveling Salesman Problem (TSP). The aim of the Traveling Salesman Problem is to visit exactly once each of selected network points (nodes, customers) and return to the point where the trip has begun. The traveling costs between each pair of nodes are known. The aim is to plan a traveling salesman trip in such a way, that he could visit every node exactly once and the total trip cost was minimal [1], [2]. In its simplest form the Vehicle Problem Routing is different from the Traveling Salesman Problem with additional constraints related to the number of vehicle and the vehicles capacity. The Vehicle Routing Problem (VRP) is NP-hard, which means that there are no exact approaches that are able to solve the real instances in acceptable time. Complexity of vehicle routing problem is of O(n-1)! where n is the number of points that should be visited. Calculations time compared to the number of data (in this case, the number of points visited by vehicles) increases exponentially.

Due to the relatively long calculations duration scientists focus on finding approximate algorithms (heuristics) that would provide acceptably good results in acceptable time. One of the methods to achieve this goal is to reduce the complexity of the vehicle routing problem. In the literature, this objective is achieved through the use of so-called two-phase methods:

- Cluster first/route second,
- Route first/cluster second.

The term "cluster" in this case refers to a subsets separated from the set of service points (customers). Each subset is operated by a single vehicle. Thus, the number of solutions decreases with increasing number of clusters. If the problem with 20 cities is divided into 5 clusters, every vehicle is visiting only 4 cities. For such number of cities there are only 24 possible vehicle routes in every cluster. This kind of the points division has its reflection in the real life transportation problems, especially when customers have at their disposal different goods groups which have different transportation susceptibility and can't be transported together. It means that some goods cannot be transported together on the same vehicle. This case concerns for example chemical companies, which produce a range of goods often requiring the use of special vehicles, because not all products may be transported on the same vehicle. Thus the vehicle used for transport can service only these customers whose product range due to its specificity may be transported on the same vehicle.

There are 3 main groups of algorithms in literature on vehicle routing problem that refer to the method cluster first/route second [3]:

- Basic clustering algorithms;
- Methods using branch & bound;
- Methods using petal algorithm.

Basic clustering algorithms perform a simple division of the service points into clusters and vehicles routing in these clusters. In branch & bound algorithm authors do not focus on creating clusters of customers, but they create the sets of feasible vehicle routes. Every level in the search tree states a feasible solution. In Petal algorithm feasible vehicle routes are created instead of the clusters. Final route selection is performed with the usage of the set partitioning problem.

For the route first/cluster second method, the first stage is to create one big route for all service points (customers), without taking into account any constraints. In the second stage this route is divided into shorter routes taking into account the constraints (there are no constraints concerning the number of vehicles). New, shorter routes are every time feasible.

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Methods for solving the vehicle routing problem, has been under the scientists research for several decades. So far, the proposed heuristic solving vehicle routing problem can be classified as follows:

- Route construction heuristics – assign customers to vehicles and determine the customer service order [4];
- Local search heuristics – replace the currently considered solution to a new one, better in terms of the final route cost. Depending on the type of algorithm this exchange may involve the exchange of the points of a given route, the exchange of the sequences of arcs between the points, the transfer of any point from one route to another [5];
- Metaheuristics (genetic algorithms, ant colony algorithms, simulated annealing, neural networks) – these are the so called overriding heuristics, controlling the process of iterative search of the lower level heuristics. A detailed review of these metaheuristics can be found in [6].

The analysis of the routing problem literature shows that there is a lack of papers describing the problem of the vehicle routing when suppliers are defined by different goods groups which have different transportation susceptibility and cannot be transported together on the same vehicle. Although the authors of [7] characterize this problem, but do not specify the problem solving methodology and do not consider very significant external transport costs, which will be discussed in chapter II.

II. External Transportation Costs

It is said that transport contributes to economic growth [8]. Unfortunately, most forms of transport do not only affect society in a positive way but also give rise to side effects. For example road vehicles contribute to congestion, trains and aircraft contribute to noise and ships contribute to air pollution. Transport users usually don't take these things into account when they make a transport decision. Therefore these effects are generally labeled as external effects. The main external transport effects consider: *congestion, accidents, air pollution, noise*. The cost associated to these effects is called the external cost. Although the estimation of external costs have to consider several uncertainties, there is a wide consensus on the major methodological issues. The best practice estimation of congestion costs is based on speed-flow relations, value of time and demand elasticity. For air pollution and noise costs, the impact pathway approach is broadly acknowledged as the preferred approach, using Values of Statistical Life based on Willingness to Pay. Marginal accident cost can be estimated by the risk elasticity approach, also using Values of Statistical Life. Given long-term reduction targets for CO_2 emissions, the avoidance cost approach is the best practice for estimating climate cost. Other external costs exist, e.g. costs related to energy dependency, but there is for the time being no scientific consensus on the methods to value them [9].

Average external costs for road freight transport including congestion, accidents, air pollution and noise were estimated for the first 17 countries of the European Union in [10]. The unit of these costs was euro/1000 ton-kilometers. The external costs were estimated for accidents, air pollution and noise. Of course authors considered the congestion costs, but they didn't estimated them. Other road freight transport costs were estimated as follows:

- Accidents — 7,6 [EURO/1000tkm],
- Noise — 7,4 [EURO/1000tkm],
- Air pollution — 42,8 [EURO/1000tkm].

III. Problem Formulation

In this paper authors present customers service optimization in the single-depot vehicle routing problem. Customers are characterized by the different goods groups. Some of these goods groups can't be transported together on the same vehicle. Optimization criterion, except the order of customer service, are the external transport costs. Considered problem assumes that every customer has only one goods group. Goods volume at every customer and possible groups connections are known. Mathematical problem formulation is an extension of the formulation proposed in [7], developed by an additional element of external transport costs.

Determining all the possible combinations of groups of goods connections requires the use of *Stirling Numbers Of The Second Kind* [11]. They count the number of ways to partition a set of n elements into k nonempty subsets. They are marked by $\left\{ {n \atop k} \right\}$. These numbers satisfy a recursive relationship such as: $\left\{ {n \atop k} \right\} = k \left\{ {n-1 \atop k} \right\} + \left\{ {n-1 \atop k-1} \right\}$ under the assumptions such as: $\left\{ {n \atop 1} \right\} = 1 \ \wedge \ \left\{ {n \atop n} \right\} = 1$ More over it is assumed that, if $k > n$, then $\left\{ {n \atop k} \right\} = 0$

Let us assume that we transport n-types of goods from which we can distinguish two goods groups: first of size t and a second of size s, which cannot be connected during the transport. Then there exists a set $\boldsymbol{P}$ of possible goods groups division into two subsets (L is a cardinality of a set $\boldsymbol{P}$), such that constraints of not connecting some goods groups during the transport are satisfied.

$$L = \left\{ {n-t-s+2 \atop 2} \right\} - \left\{ {n-t-s+1 \atop 2} \right\}$$

Example: Let us assume that we are supposed to transport five types (groups) of goods {1, 2, 3, 4, 5} where goods of type (group) 3 and 4 cannot be transported together with goods of type 1. In this situation we can define four possible divisions of goods groups set into two subsets, such that constraints of not connecting some goods groups during the transport are satisfied.

$$L = \begin{Bmatrix} 5-2-1+2 \\ 2 \end{Bmatrix} - \begin{Bmatrix} 5-2-1+1 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 4 \\ 2 \end{Bmatrix} - \begin{Bmatrix} 3 \\ 2 \end{Bmatrix} = 7-3=4$$

{1, 2, 3, 4, 5}={1, 2, 5}∪{ 3, 4} variant I
{1, 2, 3, 4, 5}={1, 2}∪{3, 4, 5} variant II
{1, 2, 3, 4, 5}={1, 5}∪{2, 3, 4} variant III
{1, 2, 3, 4, 5}={1}∪{2, 3, 4, 5} variant IV

In order to determine the order of customer service, taking into account the goods and external transport costs the cluster first/route second heuristic is used. For the purpose of determining clusters of customers, authors developed the clustering algorithm which is a modification of the algorithm presented in [12]. Every cluster generated with the usage of this algorithm contains customers with goods groups that can be transported together. More over vehicle route in every cluster is always feasible.

For a correct understanding of the algorithm it is necessary to clarify indicators appearing in this algorithm. These indicators are defined as follows:

PO – (1, 2, …*i*, *i*'…, *I*) – a list of customers,
PO' – (2, …*i*, *i*'…, *I*) – new list of customers,
P – (1, 2, …*p*,…, *P*) – list of vehicles,
L – temporary list of customers,
Q^p – vehicle's capacity,
ZP^p – vehicle's fuel usage,
$d_{i,i'}$ – distance between customers,
q_i – customer's demand,
q_k – cluster's demand,
K – cluster,
t_i – travel time between customer,
$t_{i,i'}$ – customer service time,
T^p – max. allowed driver's working time.

The clustering procedure contains the following steps:

1) Set a list of customers *PO* and sort them ascending by their distance to the depot
2) Set a list of vehicles *P* and sort the by vehicle's capacity/fuel usage Q^p/ZP^p
3) Set the maximum distance *Dmax* between customers in the cluster
4) Set a list *L*. Take the first vehicle from the list *P* and connect it with new cluster *K*
5) Take the first customer i from list PO and place it at the end of list L. Set the cluster parameters as follows:
$$q_k = q_i$$
6) Delete customer *i* from the list *PO* and save the list as *PO'*
7) Take the first customer *i'* from the list *PO'* and check if the goods of customer *i*' can be transported together with goods of customer *i*.

if not, delete *i'* from the list *PO'* and repeat step 7)

8) Check the vehicle's capacity constraints
$$q_k = q_i + q_{i'} \leq Q^p$$

if not, delete *i'* from the list *PO'* and come back to step 7)

9) Check the distance constraints
$$d_{i,i'} \leq Dmax$$

if not, delete *i'* from the list *PO'* and come back to step 7)

10) Check the drivers working time constraints
$$t_{d,i} + t_i + t_{i,i'} + t_{i'} + t_{i',d} \leq T^p$$

where *d* is a depot,

if not, delete *i'* from the list *PO'* and come back to step 7)

11) Place customer i' at the end of list *L*. Set the cluster parameters as follows: $q_k = q_i + q_{i'}$

If *PO'* is empty, save the list *L*. If not, come back to step 7)

Repeat steps 4)-11) until *PO'* is empty.

Routes optimization in every cluster is the performed with the usage of the genetic algorithm.

To build that algorithm the path representation of the vehicle routes was used. Crossover is realized with the OX method [13]. Mutation uses one of the simplest local search heuristic called 2-opt.

IV. Mathematical Problem Formulation

It is assumed that transportation network structure consists of customers, depot, and road connections. Every customer is characterized by a demand of a single goods group. Depot is equipped with vehicles of different capacity. Sets of network nodes, goods groups and vehicles are defined as follows:

C= {1, 2, …*i*, *i'*, …*I*} – set of customers.
D= {*d*} – set of depots.
GP= {1, 2, …, *gp*, ..., *gp'*,…,*GP*} – a set of goods groups.
V= {1, 2,…, *v*,…, *V*} – a set of vehicles used for transport.

In order to characterize goods groups due to the possibility of their common transport, we assume that the cartesian product of the ***GP*** set elements represents function determining potential possibility of transporting goods groups together:

$$\alpha: \quad \boldsymbol{GP} \times \boldsymbol{GP} \longrightarrow \{0, 1\},$$

where if α(*gp*,*gp*')=1,then the load *gp* can be transported together with load *gp'* (*gp*, *gp'*∈*GP*), else α(*gp*,*gp*')=0.

P= {1, 2, …,*l*, …,*L*} – a set of the goods groups divisions number due to the constraint of not connecting some goods groups during the transport.

P^l- *l*-division number of a set ***GP*** into subsets $\boldsymbol{P}_1^l$ and $\boldsymbol{P}_2^l$
$\boldsymbol{GP}=P^l=\boldsymbol{P}_1^l \cup \boldsymbol{P}_2^l$ while $\boldsymbol{P}_1^l \cap \boldsymbol{P}_1^l = \emptyset$
$\boldsymbol{P}_1^l = \{gp: \alpha(gp, gp') = 1; gp, gp' \in \boldsymbol{GP}\}$
$\boldsymbol{P}_2^l = \{gp: \alpha(gp, gp') = 1; gp, gp' \in \boldsymbol{GP}\}$ while
$\exists gp \in \boldsymbol{P}_1^l, \exists gp' \in \boldsymbol{P}_2^l$ for $\alpha(gp, gp') = 0$

Taking the type of the vehicle routing problem proposed in this paper into account, each customer is characterized by the quantity of goods in each goods group, assuming that each customer can have only one goods group q^i_{gp}.

Optimization task formulation requires characteristics defined as follows:

$\boldsymbol{D} = [d_{i,i'}]$ – matrix of distances between each node of a transportation network structure.
$\boldsymbol{T} = [t_{i,i'}]$ – matrix of travel times between each node of a transportation network structure.
t_i^v – vehicle *v* unloading time at customer *i*.
T^v – working time of a vehicle's *v* driver.
k^v – unitary cost for vehicle's *v* usage by a unit of a distance made by vehicle *v*.
kt^v – unitary cost of driver's working time [zł/min].
Q^v – vehicle's *v* capacity.
$Kz = [Kz^v]$ – external transportation costs.

Decision variable relates to the sequence of customers service by a give vehicle and is defined as follows:

$$x_{i,i'}^{v,gp} = \begin{cases} 1, \text{ if goods of group } gp \text{ are transported between} \\ \quad i \text{ and } i' \text{ by a vehicle } v \\ 0, \text{ in other case} \end{cases}$$

Regarding customers, vehicles characteristics and decision variables characteristics, the point is to obtain a minimum of the objective function defined as follows:

$$F(X) = \min \left\{ F(X)_{P^l} \right\}$$

where $F(X)_{P^l}$ for $\forall l \in \boldsymbol{P}$

$$F(X)_{P^l} = \left(\begin{array}{l} \sum\limits_{gp \in P_1^l} \left(\begin{array}{l} \sum\limits_{i \in C \cup \{d\}} \sum\limits_{i' \in C \cup \{d\}} \sum\limits_{v \in V} x_{i,i'}^{v,gp} d_{i,i'} k^v + \\ \sum\limits_{v \in V} kt^v \left[\begin{array}{l} \sum\limits_{i \in C \cup \{d\}} \sum\limits_{i' \in C \cup \{d\}} t_{i,i'}^v x_{i,i'}^{v,gp} + \\ \sum\limits_{i \in C} t_i^v \sum\limits_{i' \in C \cup \{d\}} x_{i,i'}^{v,gp} \end{array} \right] \end{array} \right) + \\ + \sum\limits_{gp \in P_2^l} \left(\begin{array}{l} \sum\limits_{i \in C \cup \{d\}} \sum\limits_{i' \in C \cup \{d\}} \sum\limits_{v \in V} x_{i,i'}^{v,gp} d_{i,i'} k^v + \\ \sum\limits_{v \in V} kt^v \left[\begin{array}{l} \sum\limits_{i \in C \cup \{d\}} \sum\limits_{i' \in C \cup \{d\}} t_{i,i'}^v x_{i,i'}^{v,gp} + \\ \sum\limits_{i \in C} t_i^v \sum\limits_{i' \in C \cup \{d\}} x_{i,i'}^{v,gp} \end{array} \right] \end{array} \right) + \\ + \sum\limits_{i \in C \cup \{d\}} \sum\limits_{i' \in C \cup \{d\}} \sum\limits_{gp \in P^l} \sum\limits_{v \in V} x_{i,i'}^{v,gp} d_{i,i'} K z^v \end{array} \right) \rightarrow \min$$

Constraints imposed on the values of decision variables are defined as follows:

- Each customer i can be serviced only once

$$\forall v \in \boldsymbol{V} \quad \forall l \in \boldsymbol{P} \quad \left(\begin{array}{l} \forall gp \in \boldsymbol{P_1^l} \sum\limits_{i' \in C \cup \{d\}} \sum\limits_{v \in \boldsymbol{V}} x_{i,i'}^{v,gp} = 1 \\ \qquad \wedge \\ \forall gp \in \boldsymbol{P_2^l} \sum\limits_{i' \in C \cup \{d\}} \sum\limits_{v \in \boldsymbol{V}} x_{i,i'}^{v,gp} = 1 \end{array} \right)$$

- Number of vehicles servicing customers cannot exceed the number of suppliers

$$\sum_{i' \in C} \sum_{v \in V} x_{d,i'}^{v,gp} \le V$$

- The trip of a vehicle v begin and ends in a depot

$$\forall v \in \boldsymbol{V}, \forall l \in \boldsymbol{P}, \forall i' \in \boldsymbol{C} \left(\begin{array}{l} \sum\limits_{gp \in \boldsymbol{P_1^l}} \left(\sum\limits_{i \in C \cup \{d\}} x_{i,i'}^{v,gp} - \sum\limits_{i \in C \cup \{d\}} x_{i',i}^{v,gp} \right) = 0 \\ \qquad \wedge \\ \sum\limits_{gp \in \boldsymbol{P_2^l}} \left(\sum\limits_{i \in C \cup \{d\}} x_{i,i'}^{v,gp} - \sum\limits_{i \in C \cup \{d\}} x_{i',i}^{v,gp} \right) = 0 \end{array} \right)$$

- The capacity of a vehicle v cannot be exceeded

$$\forall l \in \boldsymbol{P}, \forall v \in \boldsymbol{V} \left(\begin{array}{l} \forall gp \in P_1^l \ \sum\limits_{i \in C} q_i^{gp} \cdot \sum\limits_{i' \in V} x_{i,i'}^{v,gp} \le Q^v \\ \qquad \wedge \\ \forall gp \in P_2^l \ \sum\limits_{i \in C} q_i^{gp} \cdot \sum\limits_{i' \in C} x_{i,i'}^{v,gp} \le Q^v \end{array} \right)$$

- Each vehicle route duration cannot excide driver's allowed working time

$$\forall l \in \boldsymbol{P}, \forall v \in \boldsymbol{V}$$

$$\left(\begin{array}{l} \forall gp \in P_1^l \ \sum\limits_{i \in C} \left(\sum\limits_{i' \in C \cup \{d\}} x_{i,i'}^{v,gp} \cdot t_{i,i'} \right) + \left(\sum\limits_{i \in C} t_i \cdot \sum\limits_{i' \in C \cup \{d\}} x_{i,i'}^{v,gp} \right) \le T^v \\ \qquad \wedge \\ \forall gp \in P_2^l \ \sum\limits_{i \in C} \left(\sum\limits_{i' \in C \cup \{d\}} x_{i,i'}^{v,gp} \cdot t_{i,i'} \right) + \left(\sum\limits_{i \in C} t_i \cdot \sum\limits_{i' \in C \cup \{d\}} x_{i,i'}^{v,gp} \right) \le T^v \end{array} \right)$$

V. COMPUTATIONAL RESULTS

For the purpose of researches authors prepared a simple example of a distribution system that consists of a single depot and 40 customers (Fig. 1).

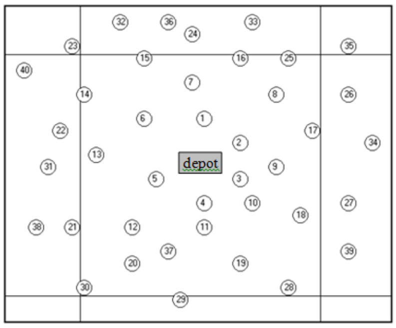

Figure 1. Distribution system structure considered in optimization example.

To illustrate the size of this system structure, the distance between customers number 39 and 40 is around 31 kilometers.

Customers are characterized by the same demand (single number of pallets) in 4 different goods groups (every customer has only one goods group). Moreover it is assumed that goods of group 1 cannot be transported together with goods of group 4. Other characteristics required for the calculations are:

- customer service time – 15 min.,
- vehicle speed – 60 km/h,
- driver cost – 0,4 zl/min.,
- max allowed driver working time – 9 h,
- vehicle (type 1) usage cost (fuel usage) – 1,5 zł/km,
- vehicle (type 2) usage cost – 1,3 zł/km,
- vehicle (type 1) capacity – 22 pallets (pallet – 1 t),
- vehicle (type 2) capacity – 17 pallets,
- external costs by 1 tkm – 0,23 zł,
- (((42,8€+7,6€+7,4€)/1000tkm)*4zł) = 0,23 zł),
- 1 € = 4 zł.

To obtain the minimum of the criteria function the author's computer application was used. Application uses clustering procedure to create feasible clusters and genetic algorithm for routing. Optimization results are presented in Table I.

TABLE I. OPTIMIZATION RESULTS

Vehicle 1 (type 1)		Vehicle 2 (type 2)		Vehicle 3 (type 2)	
Custo-mers	Goods group	Custo-mers	Goods group	Custo-mers	Goods group
27	1	30	2	23	2

9	3	21	3	32	3	
26	2	12	4	36	2	
17	3	38	2	16	4	
37	3	22	4			
28	3	40	4			
29	1	14	4			
4	3	15	2			
5	2	11	2			
25	1	18	4			
8	1	19	3			
7	2	20	2			
24	3	13	3			
6	1	31	4			
2	3					
10	2					
39	1					
1	1					
34	1					
33	1					
35	3					
3	2					
209,28 km		122,48 km		39,53 km		Route length
526,71 zł		290,424 zł		86,24 zł		Route cost
212,8 zł		131,2 zł		38,8 zł		Driver cost
100 %		82 %		23 %		Vehicle fulfilment
1058,95 zł		394,38 zł		36,36 zł		External cost

VI. CONCLUSIONS

Despite the relatively large number of customers the computer application run on 2,0 GHz CPU, has returned the optimization results within a few second.

As it is shown in Table I, optimization results are correct. Whole customers' demand is satisfied. Goods of groups 1 and 4 are not transported together. Vehicles' fulfillment is very high except the third vehicle, which had to be used due to the limited allowed driver's working time. If there was no limit to the drivers working time, customers would be serviced by 2 vehicles of the first type. The better transportation service is planned the shorter routes length and duration are, what directly influences the external transport costs.

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