Modeling and Control of an Isolated Intersection via Hybrid Petri Nets

Bassem Sammoud
Laboratoire de Recherche en Automatique (LA.R.A.), ENIT, BP 37, 1002 Tunis Belvédère
Laboratoire Systèmes et Transport (SET), UTBM, Site de Belfort, 90010 Belfort Cedex, France
Email: bassem.sammoud@utbm.fr

Mahjoub Dridi and Abdallah El Moudni
Laboratoire Systèmes et Transport (SET), UTBM, Site de Belfort, 90010 Belfort Cedex, France
Email: mahjoub.dridi@utbm.fr

Mohamed Benrejeb
Laboratoire de Recherche en Automatique (LA.R.A.), ENIT, BP 37, 1002 Tunis Belvédère

***Abstract*—This paper focuses on the use of Hybrid Petri Nets (HPNs) to model and control the traffic of an isolated intersection of four phases. It captures important aspects of the flow dynamics in urban networks.**
In this paper, we show how this model can be used to describe the traffic functioning and to obtain control strategies that improve the traffic flow at intersections. The proposed approach can be generalized to control several connected intersections in a distributed way.

***Index Terms*—transportation, Hybrid Petri Net, traffic signal control, modeling, signalized intersection.**

I. Introduction

With the growing number of vehicles, the traffic congestion and transportation delay on urban arterials are increasing worldwide; hence, it is imperative to improve the safety and efficiency of transportation.

The control strategies consist of changing the intersection's stage specification, the relative green duration of each stage, the intersection's cycle time, and the offset between cycles for successive intersections, while requiring the well-planned synchronization, schedule and control to achieve satisfactory performance.

In this work, traffic signal control systems are considered to be an hybrid system, including both continuous time and discrete event components.

There is a vast amount of literature focused on finding the modeling of an intersection [1]-[3].

For the traffic flow model based on a discrete Petri Net (PN), Wang et al. proposed a Petri Net (PN) model that includes both traffic signal control logic and traffic flow [4]. In [5], List had described how PN can be used to model traffic control situations in urban networks.

On the other side, the model based on a continuous Petri net is suitable for macroscopic description, [6] and [7]. And recently, hybrid systems have received much attention [8] and [9] and have been applied for the modeling and control of transport systems [10].

In general, the traffic control is characterized by a continuous conduct of the road [6] and behavior discrete at road crossing [1], [11]. And, these two behaviors are respectively modeled, by a Continuous Variable speed Petri Net (CVPN) and Timed Petri Net (TPN).

In order to complete these previous works, we present, in this paper, a new approach to model and to simulate the traffic signal control based on a Hybrid Petri Net HPNs method.

This paper is organized as follows: after the introduction of section 1, we present, in section 2, HPN. Then, we give, in section 3, a description of the proposed model and the control strategy of traffic light by using HPN. Section 4 describes the case study and reports the results of some simulations performed under different traffic scenarios.

II. Hybrid Petri Nets

In the present work, a signalized intersection with its input and output flows is considered to be an hybrid system thereby contains continuous places and transitions (C-places and C-transitions) and discrete places and transitions (D-places and D-transitions). The reader can find a more detailed presentation of HPNs in [9].

Definition: A marked autonomous hybrid Petri net is a sextuple $R = \langle P, T, \text{Pre}, \text{Post}, m_0, h \rangle$ fulfilling the following conditions

$P = \{P_1, P_2, ..., P_n\}$ is a finite, not empty, set of places;

$T = \{T_1, T_2, ..., T_m\}$ is a finite, not empty, set of transitions;

$P \cap T = \emptyset$, i.e. the sets P and T are disjointed;

$h: P \cup T \rightarrow \{D, C\}$, called hybrid function, indicates for every node whether it is a discrete node (sets P^D and T^D) or a continuous one (sets P^C and T^C);

Pre: $P \times T \rightarrow Q_+$ or N is the input incidence application;

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Post: $P \times T \rightarrow Q_+$ or N is the output incidence application;

m_0: $P \rightarrow R_+$ or N is the initial marking.

In the definitions of Pre, Post, and m_0, N corresponds to the case where $P_i \in P^D$, and Q_+ or R_+ corresponds to the case where $P_i \in P^C$.

Pre and Post functions must satisfy the following criterion: if P_i and T_j are such that $P_i \in P^D$ and $T_j \in T^C$, then $\text{Pre}(P_i,T_j) = \text{Post}(P_i,T_j)$ must be verified.

An unmarked hybrid PN is a 5-uple Q = <P, T, Pre, Post, h>, i.e., in addition to the structure, the nature of each node (discrete or continuous) is specified in Q.

In a continuous PN, the enabling degree of transition T_j for marking m, is defined as:

$$enab(T_j, m) = \min_{P_i \in T_j} \left(\frac{m(P_i)}{\text{Pr}\,e(P_i, T_j)} \right) \tag{1}$$

A discrete transition in a hybrid PN is enabled if each place P_i in T_j meets the condition (as for a discrete PN)

$$m(P_i) \geq \text{Pr}\,e(P_i, T_j) \tag{2}$$

Hybrid Petri nets (HPNs) were initially successfully applied for modeling, performance evaluation and the manufacturing systems design and more recently they had also been successfully used for studying transportation systems, [12].

Graphically for HPN, the arrows represent the arcs, discrete places are represented by circles and discrete transitions are represented by black boxes, whereas continuous places are represented by double circles and continuous transitions are represented by white boxes.

III. Modeling a Signalized Traffic Intersection by HPNs

A. *Studied Intersection Description*

The studied configuration is related to a crossroad that has an intersection of four directions, with two links, one input link formed of 2 lanes with movements to turn left and to go straight or right, and one output link. An example of this signalized intersection is given in Fig. 1.

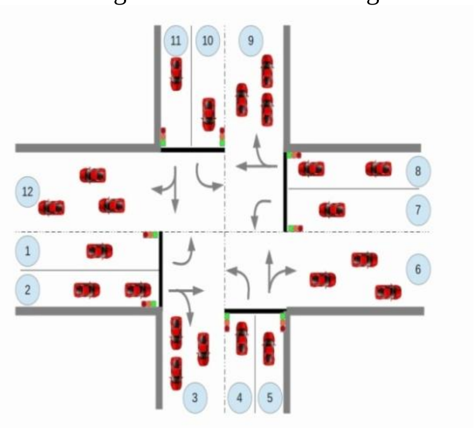

Figure 1. A four phases signalized intersection

We consider here what we call a high type of intersections, where all traffic flow conflicts are separated by signal phasing. In this kind of intersection, there is four phases as shown in Fig.2.

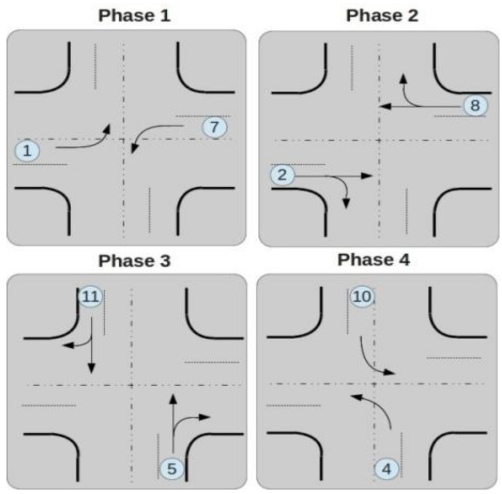

Figure 2. Four phases intersection

In order to prevent the conflicts, we can use the notion of compatible streams and incompatible streams as presented in [13]. Streams, which can simultaneously get the right-of-way, are called compatible streams for example streams 1 and 7. On the other hand, when trajectories of two traffic streams do cross, the streams are in conflict, for example, streams 1 and 11.

In our case, the studied intersection can be partitioned into four Compatible Stream Groups (CSG) as shown in Fig 2. CS 1: stream 1 and 7; CS 2: stream 2 and 8; CS 3: stream 5 and 11; CS 4: stream 4 and 10

B. *Analytic Model of Road Section*

In this part, we present the model by CVPN to model the flow of traffic on the ways of the crossroad.

To model the problem, we divide each section into N segments $[X_i, X_{i+1}]$ of length Δ_i (Fig.3). We assume that for each segment the characteristic variables of flow are supposed to depend only on the time and not on the position, like in [10].

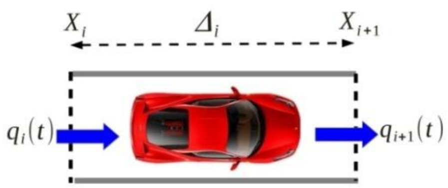

Figure 3. The generic road section

The following net of Fig 4 is composed of two continuous places (P_j and P_{j+1}). The marking (m_i) of place P_i describes the number of vehicles which enter the intersection and the marking ($m_i^{'}$), such that $m_i^{'} = c_i - m_i$, describe the available places on the i$^{\text{th}}$ stream, and the markings a_j and a_{j+1} describe the number of possible simultaneous firings for transitions T_i and T_{i+1}. Moreover, a discrete PN is used to represent the traffic light.

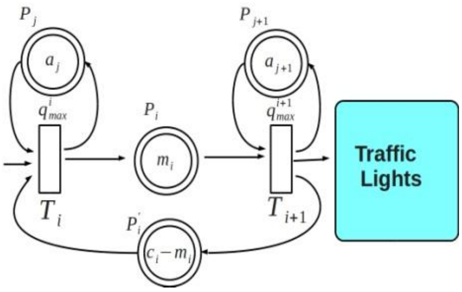

Figure 4. A hybrid PN model for a road intersection

It should be noted that the sum of the markings of places P_i and P'_i (*i.e.* $m_i + m'_i$) is an invariant marking. It is the same for P_j places (i.e. m_j), which remains constant over time. c_i represents the capacity of each way. We have

$$\begin{cases} m_i(t) + m'_i(t) = c_i & \forall t \geq 0 \\ m_j(t) = a_j & \forall t \geq 0 \end{cases} \quad (3)$$

The evolution steps of CVPN [David et al 1992] can allow us to write the following mathematical expressions defining the speed crossing $q_i(t)$ and $q_{i+1}(t)$ related to the transitions T_i and T_{i+1} respectively

$$\begin{cases} q_i(t) = q_{\max i} \min(a_j, m'_i(t)) \\ \quad = q_{\max i} \min(a_j, c_i - m_i(t)) \\ q_{i+1}(t) = q_{\max i+1} \min(a_{j+1}, m_i(t)) \end{cases} \quad (4)$$

We define the $q_{\max i}$ as the maximal firing frequency of transition T_i. And the marking m_i of place P_i is given by the following equation

$$\frac{dm_i(t)}{dt} = q_i(t) - q_{i+1}(t) \quad (5)$$

The marking of a place is represented by a continuous positive real number. However, those of a discrete place are non-negative integer. The crossing of a discrete transition may affect the marking of continuous PN and vice versa.

The evolution of the CVPN is also given by the system of differential equations (see the system of equation (5) and (6) or (7) or (8)). We observed that there are several phases in this evolution (we call a phase, one period of time during which the expression speeds according to markings remains unchanged). We pass from one phase to the following, when the marking of a place *Pi* crosses the threshold m_i=1, in a direction or in the other.

We can distinguish 3 cases according to the speed crossing $q_i(t)$ and $q_{i+1}(t)$.

- A first case, called exponential phase, will last as long as $a_i<c_i-m_i$ and $a_i>m_i$

$$if \quad a_i < c_i - m_i \quad and \quad a_i > m_i \ then\,(5)\,with$$
$$\begin{cases} q_i(t) = q_{\max i} \\ q_{i+1}(t) = q_{\max i+1} m_i(t) \end{cases} \quad (6)$$

- When marking m_i reaches value 1, we pass to a second phase (case 2), labelled linear phase and called so because the evolution is linear, is given by

$$if \quad a_i < c_i - m_i \quad and \quad a_i < m_i \ then\,(5)\,with$$
$$\begin{cases} q_i(t) = q_{\max i} \\ q_{i+1}(t) = q_{\max i+1} \end{cases} \quad (7)$$

- And a second exponential phase (case 3) which happens when *ci-mi=1*

$$if \quad a_i > c_i - m_i \quad and \quad a_i < m_i \ then\,(5)\,with$$
$$\begin{cases} q_i(t) = q_{\max i}(c_i - m_i(t)) \\ q_{i+1}(t) = q_{\max i+1} \end{cases} \quad (8)$$

The resolution of differential equations, associated with each case, depends on the initial conditions, on initial time t_0 and initial condition m^0_i, leads to defining the following markings equations

$$Case\,1:\ a_i < c_i - m_i \quad and \quad a_i > m_i$$

$$m_i(t) = \frac{q_{\max i}}{q_{\max i+1}} + \left[m_i^0 - \frac{q_{\max i}}{q_{\max i+1}} \right] e^{-q_{\max i+1}(t-t_0)} \quad (9)$$

$$Case\,2:\ a_i < c_i - m_i \quad and \quad a_i < m_i$$

$$m_i(t) = (q_{\max i} - q_{\max i+1})t + m_i^0 \quad (10)$$

$$Case\,3:\ a_i > c_i - m_i \quad and \quad a_i < m_i$$

$$m_i(t) = \frac{q_{\max i} c_i - q_{\max i+1}}{q_{\max i}} + \left[m_i^0 - \frac{q_{\max i} c_i - q_{\max i+1}}{q_{\max i}} \right] e^{-q_{\max i}(t-t_0)} \quad (11)$$

C. The Proposed HPNs Model

In this section, the Hybrid PN of the intersection, given in Fig.1 is described with the aim to provide an example of a HPN model of an urban transportation system. For this optic, we use the model presented in Fig 4, and the overall proposed HPN model is given in Fig 5, and it will be generalized later for a common configuration. The proposed HPN model is described as follow $\forall i \in \{1,2,4,5,7,8,10,11\}$

P_{i1}: vehicles arriving on the i^th^ stream,

P_{i2}: vehicles present on the section of the i^th^ stream,

P_{i3}: available places on the i^th^ stream,

P_3, P_6, P_9, P_{12}: the output links,

a_j: simultaneous firing parameters of transitions T_{i1} and T_{i2}. (a_j= 2 vehicles and a_{j+1}= 1 vehicle).

The traffic lights in our discret PN model are presented by places and transition where

P_{ij}: green period for streams $\{i, j\}$,

T_{ij}: green phase (with d_{ij} duration) ends for streams {*i*, *j*}.

In this paper, we simulate the evolution of traffic in an isolated intersection where the traffic lights switch from the usual order (fixed signal-timing). We define four compatible phases as a set of permitted conflicts: {1, 7}, {2, 8}, {5, 11} and{4, 10}. The considered HPN model is given in Fig.5.

In the aim to advance some ideas leading to a control strategy for urban traffic networks, we propose, a new heuristic approach to control the considered model.

Our control strategy is based on the green light duration, which is determined on the estimated number of vehicles entering the road during a cycle.

We define a cycle as a sum of green light and red light durations. We take the maximal number in each pair of compatible streams with movement to turn left and we add a value of t_d as shown in equation (12)

$$\forall(i,j) \in \{(1,7),(4,10)\} \tag{12}$$

$$d_{ij} = \max(Nbv_i, Nbv_j)t_c + t_d \tag{13}$$

Nbv_i is the number of vehicles in link i , t_d the starting time of the vehicle at the intersection and t_c the crossing time.

As an example, we assume that the numbers of vehicles in streams 7 and 1 are respectively 5 and 8. So the green duration is computed by $d_i = 8 + t_d$.

Since the maximal firing frequency to the rate of output is 1 veh/s. So, let consider t_d=t_c=1s, as well for the flows 4 and 10.

In the other side, as cited previously, the arrival flow of vehicles for {2, 5, 8, 11} is more important than the ones with movements to turn left, so the green duration is computed as follows: we divide the number of vehicles entering in the intersection by the number of roads (i.e. equation (13)).

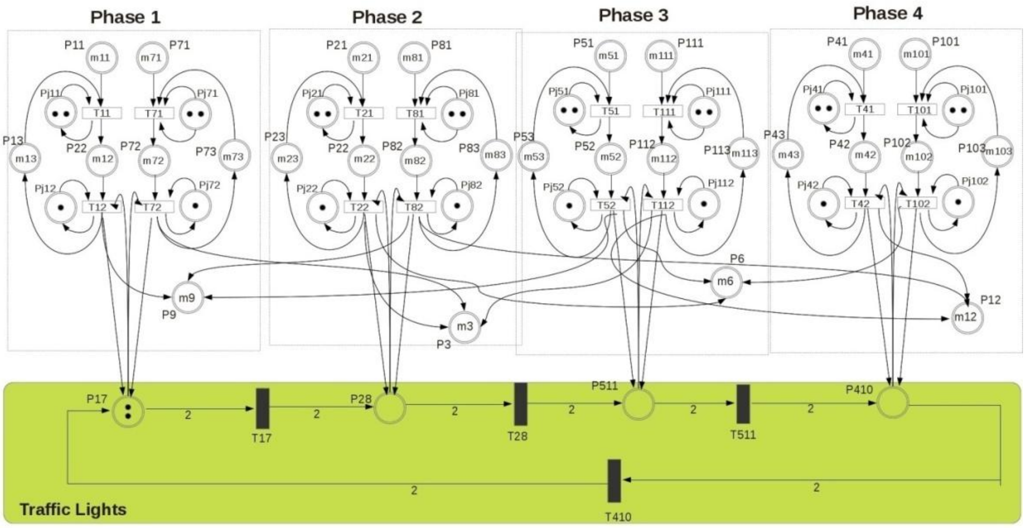

Figure 5. The proposed hybrid PN intersection

with

IL: the number of input links,

Nbv_k: the number of vehicles in each link,

t_d: the starting time of the vehicle at the intersection,

t_c: the crossing time.

IV. Simulation and Results

As follows, we suppose that the arrival rate of vehicles is Nb_Vehicles divided randomly in different stream (see Table I).

with

Nbv: arrival rate of vehicles of stream i,

TEi: evacuation time of all vehicles for each flow,

d_{ij}: green light duration.

During the simulations, we considered that the maximal firing frequency for T_{i1} is equal to 2veh/s; on the other side, the rate of output (for T_{i2}) is 1veh/s and we considered intersection currently ruled by a fixed signal-timing plan.

TABLE I. Simulation Parameters 1

Stream i	*Nbv i*	Sim 1		Sim 2		Sim 3 (case a)		Sim 3 (case b)		Sim 4	
		d_{ij}	TE1	d_{ij}	TE2	d_{ij}	TE3	d_{ij}	TE4	d_{ij}	TE6
1	5	5	18	7	4	10	4	10	4	9	5
2	20	5	78	10	72	15	64	20	67	13	58
4	7	5	33	7	31	10	45	10	53	11	40
5	15	5	62	10	52	15	39	20	43	13	34
7	8	5	20	7	35	10	9	10	8	9	8
8	20	5	76	10	49	15	63	20	29	13	58
10	10	5	35	7	62	10	89	10	106	11	44
11	25	5	99	10	85	15	83	20	98	13	74

At first step, we suppose that Nb_Vehicles=110 and the delay d_{ij} associated to the discrete transition (green light duration), is the same and fixed to 5 seconds (Sim1). The evolution of markings is shown in Fig 6.

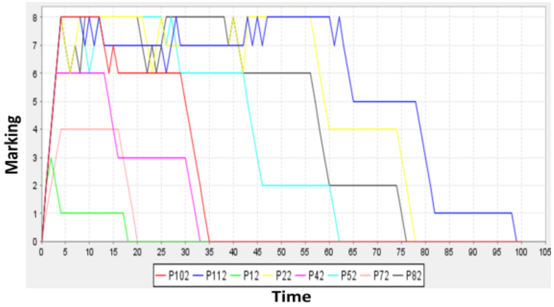

Figure 6. Marking evolution (Sim 1).

In the second step, we do not consider a same delay d_{ij} for all the streams, and we distinguish three types of simulation (Sim 2, Sim 3 cases a and b).

In the first time, we start by increasing the green light duration, i.e. 7 seconds for left turn movements (streams 1/7 and streams 4/10) and 10 seconds for the streams to go straight or right (streams 2/8 and streams 5/11). The evolution of marking is shown in Fig 7. We can see that the total evacuation time is reduced from 99 to 85 s.

In a second time, and referring to a real intersection, we can notice that the arrival rate of vehicles in the streams {1, 4, 7, 10}, are less than {2, 5, 8, 11}. For this reason, in the simulation (Sim 3) we start by fixing the green duration for the last streams to 15 in *case a* and then to 20 seconds in *case b*. The other streams are left with duration of 10 seconds. The behavior of marking for these two cases is presented in Fig 8 and Fig 9.

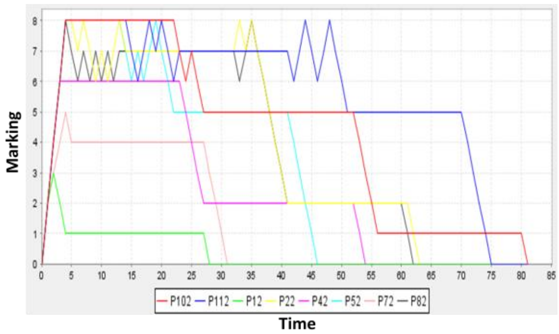

Figure 7. Marking evolution (Sim 2).

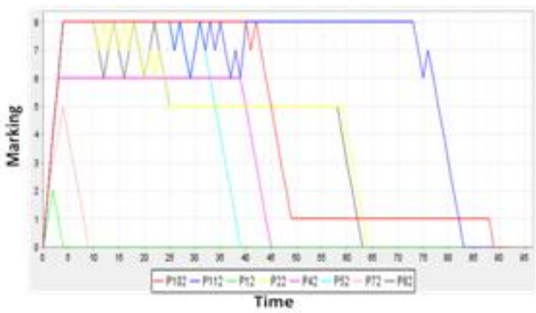

Figure 8. Marking evolution (Sim 3 (case a)).

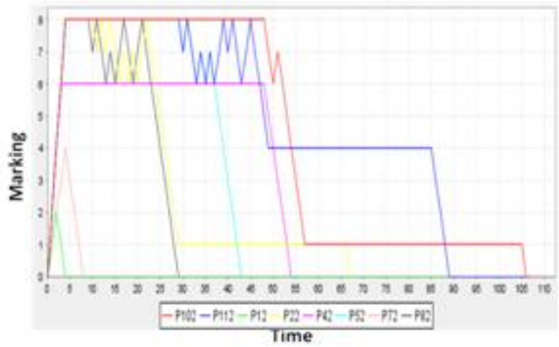

Figure 9. Marking evolution (Sim 3 (case b)).

The simulation (Sim 3) shows clearly that despite this duration seems to be directly related to the time needed to avoid all the ways; we can notice that it can not be always the case, as shown in Sim 3 (case a and case b).

In the last step, and for the last simulation, we use the proposed heuristic approach to optimize the green time duration. This approach is based on the estimated number of vehicles which enter the crossroad during a cycle (equation 10 and 11).

We obtain the marking evolution presented in Fig 10.

We can clearly see that the total evacuation time is greatly reduced, since it passes to 74s.

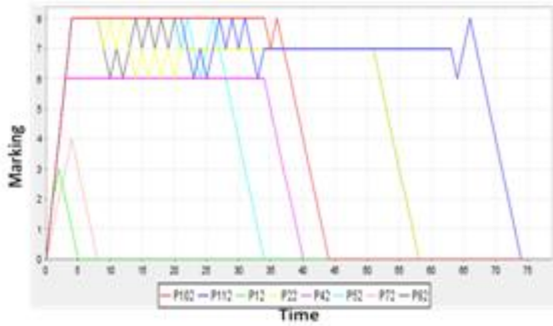

Figure 10. Marking evolution (Sim 4).

In order to prove the robustness and the efficiency of our approach, we realized other simulation by changing parameter Nb_Vehicles. These simulations are reported in Table II. We can see that our approach provides an interesting result.

In Fig. 11, we report the results of all these approaches for different flows capacities and we can see that for all the considered cases our heuristic provides the best results (Sim 4).

V. Conclusion and Future Work

This paper has presented a new hybrid model of signalized intersection. This model based on HPNs describes the behavior of the traffic with reference to a four phase signalized intersections. We partionned intersection streams into several Compatibles Streams Group (CSG) with a simultaneous movement through the intersection. Some experimental results about a considered case study were discussed in order to describe the dynamics of the model and the efficiency of the proposed heuristic. The researches continue by a development of an algorithm to decide the optimal traffic signal plan by the minimization of the overall evacuation time. Our future work will focus also on how several neighbor intersections will be modeled and controlled together.

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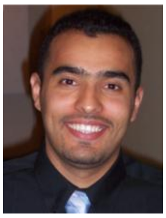
Bassem SAMMOUD was born in Tunisia in 1985. He obtained the Master of Automatic Control and Signal Processing from the "Ecole Nationale d'Ingnieurs de Tunis" in 2009. he is currently a PhD student in Automatic Control and Signal Processing in LARA-ENIT. His current research interests are the traffic control. and optimization methods for discrete events systems and operational research domains.



Mahjoub DRIDI was born in Tunisia, in 1976. He received in 2001 the M.Sc. degree in control and computer sciences from de University of Lille 1, Villeneuve d'Ascq, France. He received his Ph.D. degree in 2004 from the Ecole Centrale de Lille, France. He is currently an assistant Professor at Université de Technologies de Belfort Montbéliard in Systèmes Et Transport Laboratory. His research is related to traffic control, production planning, computer science, optimization methods for discrete events systems and operational research.

Abdellah EL MOUDNI Received a PH.D degree in Automatic Control in 1985 from Lille University, Lille, France. He is currently a Professor at UTBM, where he is in charge of courses in automatic control. His research interests include transportation systems, nonlinear control theory, dynamic systems, control of discrete event systems and singular perturbation methods in discrete time.

Mohamed BENREJEB has obtained his Diploma of "Ingénieur IDN" (French "Grande Ecole") in 1973, a Master degree of Automatic Control in 1974, a Ph.D. in Automatic Control of the University of Lille in 1976 and the DSc of the same University in 1980. He is currently a full professor at the "Ecole Nationale d'Ingénieurs de Tunis" and an invited Professor at the "Ecole Centrale de Lille". His research interests are in the area of analysis and synthesis of complex systems based on classical and non conventional approaches and recently in discrete event systems domain.