

Airline Network: Critical Leg Assessment via Variation in Practical Capacity

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Abstract—This research to the best of our knowledge is the first to quantify airline network sustainability in the presence changeable capacity of legs and alternative flights. In this article we try to recognize critical legs via changing practical capacity of airline network components. We try to assess the behavior of legs and network via variation of leg capacities while proposing a new leg cost function, also we demonstrate how to capture the robustness of airline network in the case of variable legs represented by decreasing and increasing capacities. We are using Relative Total Cost indices to assess airline network sustainability in the case of behavior associated with User–Optimization. In this article from different point of view, passenger’s rout preference behaviors are the main subject. Numerical case study is presented for illustration purposes.

Index Terms—Airline network, arc capacity variation, calibrated link cost function, Relative Total Cost indices, Network Sustainability, User–Optimization.

I. INTRODUCTION

Networks are complex, typically, large-scale systems, and their formal study has attracted much interest from a plethora of scientific disciplines [4].

As *Ortizar and Willumsen*, (2001) mentioned, The transportation system can be viewed as a conventional economic system with demand and supply subsystems. The demand side is comprised by mode specific origin destination (O-D) matrices. The supply side of a transportation system is comprised by a network represented by links, nodes and their associated costs. In traffic assignment, an O-D trip matrix is loaded onto the network and a set of link flows is produced [15].

In this paper, we propose a novel approach for evaluating the sustainability of an airline network based on the relative total cost of the transportation network in the case of leg variation captured through a uniform link capacity ratio. The relative total cost index can be evaluated at either User-Optimal (U-O) traffic flows or System-Optimal (S-O) traffic flows. A modified leg cost function enables the quantitative assessment of the changes in the relative total cost of a transportation network, in the case of alternative travel behavior, when the link practical capacities are decreased or increased.

The increase and decrease of air carriers in networks due to maintenance, scheduling and routing approaches, air planes and airports deterioration over time, as well as politic decisions lead to time consuming and costly connection flights, lack of flights and poor service quality would effect passenger decision manners. The important factors in flight selection process for passengers are fare, welfare and flight time possibility [11].

The research into the robustness of transportation networks in the presence of disruptions is relatively recent. To the best of our knowledge, the papers of Sakakibara et al. (2004) and of Scott et al. (2006) stand as the first attempts to address the robustness of transportation networks. Nagurney and Qiang (2010) provides an overview of some of the recent developments in the assessment of network vulnerability and robustness through appropriate tools that assist in the quantification of network efficiency (performance) and the identification of the importance of network components, such as nodes and links [13]. In an airline network there are hub and spokes that every leg and flight variance is affecting the whole network sustainability. In this try all the words link or arc are mentioning a flight and the word node is mentioning airport.

This paper is organized as follows. In Section 2, we propose the components of the relative total cost index. Forasmuch as the well-known U-O and S-O transportation network models corresponding, respectively, to Wardrop’s first and second principles of travel behavior (cf. Wardrop, 1952; see also, e.g., Beckmann, McGuire and Winsten, 1956, Dafermos and Sparrow, 1969, Smith, 1979, Dafermos, 1980, Sheffi, 1985, and Nagurney, 2000) We will show that, for the same network topologies and with user link cost functions, but linear, that the relative total cost index under the U-O flow pattern can be obtained via an slightly modified formula. In section 3 we describe the relative total cost index that can be used to assess transportation network robustness and which permits either U-O or S-O travel behavior. In section 4 for the first time we are trying to evaluate the airline network sustainability by the reduction and inflation of flight capacity by network robustness measure which has been established by Nagurney and Qiang. In section 5, a case

study with real data from a partial network of an airline has been considered.

II. PRINCIPAL AND COMPONENTS OF RELATIVE TOTAL COST INDICES

A. Decentralized Decision-Making and Centralized Decision-Making (U-O and S-O)

Wardrop (1952) explicitly recognized alternative possible behaviors of users of transportation networks and stated two principles, which are commonly named after him. These principles correspond, in effect, to decentralized versus centralized behavior on networks and, although stated in a transportation context, have relevance to many different network systems. Hence, we now recall Wardrop's two principles:

First Principle: The journey times of all routes actually used are equal and less than those that would be experienced by a single vehicle on any unused route.

Second Principle: The average journey time is minimal [2].

The first principle corresponds to the behavioral principle in which travelers seek to (unilaterally) determine their minimal costs of travel whereas the second principle corresponds to the behavioral principle in which the total cost in the network is minimized.

Beckmann, McGuire, and Winsten (1956) established the equivalence between the *traffic network equilibrium* conditions, which state that all used paths connecting an origin-destination pair will have equal and minimal travel times (or costs); corresponding to Wardrop's first principle, and the Kuhn-Tucker conditions of an appropriately constructed optimization problem, under a symmetry assumption on the underlying functions. Hence, in this case, the equilibrium link and path flows could be obtained as the solution of a mathematical programming problem.

Dafermos and Sparrow (1969) coined the terms *user-optimized* (U-O) and *system-optimized* (S-O) transportation networks to distinguish between the two distinct situations in which users act unilaterally, in their own self-interest in selecting their routes, and in which users select routes according to what is optimal from a societal point of view, in that the total cost in the system is minimized. In the latter problem, marginal (total) costs rather than average costs are equilibrated equalized. The former problem coincides with Wardrop's first principle, and the latter with Wardrop's second principle. In this paper we only mentioned the U-O attitude [13].

B. The Network Equilibrium (U-O) Model with Fixed Demands

Consider a general network $\mathcal{G} = [\mathcal{N}, \mathcal{L}]$, where \mathcal{N} denotes the set of nodes, and \mathcal{L} the set of directed links. Let a denotes a link of the network connecting a pair of nodes, and let p denote a path consisting of a sequence of links connecting an origin destination (O/D) pair of nodes. The paths are assumed to be acyclic, in transportation networks, nodes correspond to origins and destinations. Let P_w , denote the set of paths connecting the O/D pair of nodes w . Let P denote the set of all paths

in the network and assume that there are n_w origin destination pairs of nodes. We assume in all models that the networks are (strongly) connected, that is, that there is at least one path connecting each pair of O/D nodes [1].

Let X_p represent the nonnegative flow on path p and let f_a Denote the flow on link a . These flows, in different settings, would correspond passenger flows.

Denote the demand associated with O/D pair w , for all $w \in W$, *Assumed*, for now, as being fixed and known, where path p contains link a , $\delta_{ap} = 1$, and otherwise $\delta_{ap} = 0$ [2].

- Optimization problem for User Optimality

$$\begin{aligned} \text{Min} \quad & \sum_{a \in A} \int_0^{f_a} t_a(y) dy \cdot & (1) \\ \text{s. t.} \quad & \sum_{p \in P_w} x_p = d_w \quad , \quad \forall w \in W \\ & f_a = \sum_{p \in P} x_p \delta_{ap} \quad , \quad \forall a \in A, \\ & x_p \geq 0 \quad , \quad \forall p \in P \end{aligned}$$

The cost experienced by a user traversing flight a is denoted by $t_a(f_a)$ [12].

C. Trip Cost function

In this section we are using a developed trip cost function that is a combination of passenger flow function and arc cost function.

- We know cost of a path in an airline network is cost of legs+ cost of transshipment or hub [15].

- Also we know flows of passenger by plane a and b are f_a and f_b .

- If nominal capacity of plane a is c_a then, practical capacity of a is $lf \times c_a$ where lf is load factor [4].

- We will show flight cost or ticket price by fc .

- The rate of flow by plane number a is $\frac{f_a}{lf \times c_a}$.

But we have two types flight, direct and indirect flights (flight with transshipment or connection flights), so;

If flight with plane a is a direct flight, then our trip cost function, is

$$t_a = fc \left[\alpha \left(\frac{k f_a}{lf \times c_a} \right)^\beta \right] + \text{arc cost}, \quad (2)$$

If flight with plane b is indirect flight, then our trip cost function varies to

$$t_a = fc \left[\alpha \left(\frac{k f_a}{lf \times c} \right)^\beta \right] + \text{arc1 cost} + \text{arc2 cost} + \text{tsc} \quad (3)$$

tsc: Transshipment Cost

α , β and k : the congestion rates. (Are positive and special coefficients for every field and company)

D. Algorithms for solving Variational Inequality and Trip Assignment

1. The Projection Method
2. The Modified Projection Method
3. The Euler Method
4. Balance Algorithm

E. Performance measure of Nagurney and Qiang for Evaluating of critical arcs

The network performance / efficiency measure is as below;

$$e = e(G, d) = \frac{\sum_w \lambda_w \frac{d_w}{n_w}}{n_w} \quad (4)$$

where d_w is demand between O/D. n_w is the number of O/D pairs in the network and λ_w is minimum cost flights between O/D.

III. RELATIVE TOTAL COST INDICES

Network total cost is;

$$TC = \sum_{g \in A} \hat{t}_g = \sum_{g \in A} t_g(f_g) f_g \quad (5)$$

Let's suppose $g \in A$ is an arc on network and $\Psi(\{g\})$ is relative total cost increase of G , and if we eliminate $\{g\}$ from network, relative total cost increase is equal to:

$$Y(\{g\}) = \frac{TC(G - \{g\}) - TC(G)}{TC(G)} \quad (6)$$

Where $TC(G)$ is total cost of the network G , $TC(G - \{g\})$ is total cost of the network $G - \{g\}$.

Because of deriving total cost from U-O, we can write the following functions.

$$Y_{U-O}(\{g\}) = \frac{TC_{U-O}(G - \{g\}) - TC_{U-O}(G)}{TC(G)} \quad (7)$$

Note: Above-mentioned function will distinguish critical nodes; when one eliminates a node all of the arcs, which terminated to this node will be eliminated.

If g has been affected by capacity changes then, the relative total cost index appears as:

$$Y_{(g)}^g = \frac{TC_{(g)}^g - TC}{TC} \quad (8)$$

$$Y_{(g)}^a = \frac{TC_{(g)}^a - TC}{TC} \quad (9)$$

TC : Total cost of network, if there is not any capacity changes.

$TC_{(g)}^g$: Network total cost if capacity decrease rate is g .

$TC_{(g)}^a$: Network total cost, if increase rate of capacity is α .

IV. AIRLINE NETWORK ROBUSTNESS ASSESSMENT

A. Airline Network Robustness with Reduction Flight Capacity

To evaluate network robustness let's decrease legs (flight) carrying capacity with a fix rate. Network efficiency measures are capturing under this reduction. If original capacity of a leg is c_g and $g(g \in (0,1])$ is the

reduction rate of capacity, gc_g , is the leg's decreased capacity and, $c_g - gc_g$ is reduction or fall measurement of leg. The network of G robustness measurement is R^g [14].

$$R^g = R^g(G, d, t, c, g) = \frac{e^g}{e} \cdot 100\% \quad (10)$$

d : G demand vector

t : Flight cost function

c : Flight capacity vector

g : Flight capacity reduction rate

e : Network performance index when capacity is c

e^g : Network performance index when capacity decreased to gc

If the performance index of a network with gc capacity is approximate c , then that network is sustainable [12].

If there is only one flight between O/D; then robustness upper bound is $g^b \cdot 100\%$ and,

$$R^g = \frac{g^b [c_g^b + kd_w^b]}{g^b c_g^b + kd_g^b} \cdot 100\% \quad (11)$$

If there are more than one flight between O/D Then;

$c^o = c_a + c_b + \dots + c_n$ And lower bound is $g \cdot 100\%$,

$$R^g = \frac{gc + kgd_w}{gc + kd_w} \cdot 100\% \quad (12)$$

B. Airline Network Robustness With Inflation Of Flight Capacity

To evaluate network robustness let's increase legs (plane) carrying capacity with a fix rate. Network efficiency measures are capturing under this reduction. If original capacity of a leg is c_g and a ; ($a \geq 1$), is the inflation rate of capacity, ac_g , is the leg's increased capacity and, $ac_g - c_g$ is inflation measurement of leg (flight) [6]. The network of G sustainability or robustness measure is

$$R^a = R^a(G, d, t, c, a) = \frac{e^a}{e} \cdot 100\% \quad (13)$$

a : leg capacity inflation rate

$a^b \cdot 100\%$ is upper bound of network robustness and

$a \geq 1$, and upper bound is $a \cdot 100\%$

$$R^a = \frac{ac + kad_w}{ac + kd_w} \cdot 100\% \quad (14)$$

V. CASE STUDY

The data for a partial network of an airline in Turkey illustrated in Fig 1, there are 5 airports in 5 cities and Ankara is a Hub station. We are interested to assess this network sustainability with critical legs upon supply and demand between the nodes Istanbul-Antalya and

Istanbul-Trabzon. Load factor policy of firm averagely is 90%. Transshipment cost for every hour 7\$ per passenger. All ticket prices averagely 15\$ per path (without taxes). The other data are illustrated in Fig 1. We are interested in daily calculations. One can observe further information in appendix.



O/D	Flight count	Plane Type	Demand
IST-TRZ	5	1,3,5	1700
IST-ANK	38	1,2,3,4,8,9	12000
IST-KNY	3	2,3	1400
IST-ANT	10	1,2,5,4	4200
ANK-ANT	2	8	1150
ANK-TRZ	2	4	780
TRZ-ANT	2	6	750
KNY-ANT	1	5	330

Figure 1. The airline network with one hub and several spokes and flight data

Solving the problem:

Step 1. Using trip cost function.

Step 2. Deriving f_g via Coding of Balance Algorithm for variational inequality and trip assignment, which are using C++ (Appendix A), and MATLAB for coding of Balance algorithm and calculations.

Step 3. To identify critical paths through the results.

Definitions of Tables:

As illustrated in Fig.2, the effects of every increasing or decreasing rates in legs capacities, from total cost index is demonstrated.

For an instance, if the firm decrease the capacity of flights from IST-TRZ, users reflexes will increase the total cost amounts and this will effects ticket prices. Fig.3 illustrates the relative total cost indices with U-O and the rates of inflation and reduction of leg capacities. For example, with increasing the rate of flight capacities in IST-TRZ leg, the Relative Total Cost will decrease (from -0.026 to -0.11).

Fig.3 illustrates Network Sustainability with whole network capacity changes via U-O. If you increase the whole capacity of network by the rate of 1.6 the network robustness will improve.

TC^γ	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
IST-TRZ	3567	3590	3605	3700	3945
IST-ANK	4044	4030	4060	3840	3765
IST-KNY	4900	4900	4900	4900	4900

IST-ANT	5442	5545	5625	5664	5619
ANK-ANT	5209	5209	5199	5199	5091
ANK-TRZ	3416	3416	3416	3416	3416
TRZ-ANT	6309	6309	6309	6309	6309
KNY-ANT	2670	2670	2670	2670	2670

TC^α	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
IST-TRZ	3567	3472	3357	3158	3158
IST-ANK	4044	4190	4230	4304	4370
IST-KNY	4900	4900	4900	4900	4900
IST-ANT	5442	5012	5230	5307	5411
ANK-ANT	5209	5400	5469	5498	5502
ANK-TRZ	3416	3128	3139	3141	3260
TRZ-ANT	6309	6309	6302	6309	6309
KNY-ANT	2670	2670	2670	2670	2670

Figure 2. Total costs with U-O and the rates of reduction and inflation of leg capacities

ψ^γ	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
IST-TRZ	0	0.0065	0.011	0.037	0.1
IST-ANK	0	-0.004	0.0039	-0.05	-0.069
IST-KNY	0	0	0	0	0
IST-ANT	0	0.0005	0.033	0.04	0.032
ANK-ANT	0	0	-0.002	-0.002	-0.02
ANK-TRZ	0	0	0	0	0
TRZ-ANT	0	0	0	0	0
KNY-ANT	0	0	0	0	0

ψ^α	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
IST-TRZ	0	-0.026	-0.058	-0.11	-0.11
IST-ANK	0	0.036	0.046	0.064	0.08
IST-KNY	0	0	0	0	0
IST-ANT	0	-0.08	-0.039	-0.024	-0.005
ANK-ANT	0	0.036	0.05	0.055	0.056
ANK-TRZ	0	-0.08	-0.081	-0.08	-0.045
TRZ-ANT	0	0	0	0	0
KNY-ANT	0	0	0	0	0

Figure 3. Relative total cost with U-O and the rates of reduction and inflation of leg capacities

$\beta = 4, \alpha = 1, k = 0.15$	R^γ	$\beta = 4, \alpha = 1, k = 0.15$	R^α
$\gamma = 0$	0	$\alpha = 1$	0
$\gamma = 0.2$	0.988	$\alpha = 1.2$	1.1432
$\gamma = 0.4$	1.054	$\alpha = 1.4$	1.1328
$\gamma = 0.6$	0.941	$\alpha = 1.6$	1.1556
$\gamma = 0.8$	1.014	$\alpha = 1.8$	1.1437

Figure 4. Network sustainability measures with whole network capacity changes via U-O using (10)

Results: IST-TRZ, IST-ANK, IST-ANT, ANK-ANT and ANK-TRZ are critical legs. (Fig. 2). Reduction capacity of (IST-TRZ) is not suggested but inflation in rate 1.6 is the priority. (Fig. 2 and Fig. 3). Decreasing capacity of (IST-ANK) with rate of 0.8 is suggested. Increasing capacity of (IST-ANT) with the rate of 1.2 is firmly suggested. Changes in the capacity of (ANK-TRZ) are not logical. (Fig. 2 and Fig. 3). Decreasing capacity of (ANK-ANT) with the rate of 0.8 will be logical. (Fig. 2 and Fig. 3). Decreasing or increasing the capacity of (TRZ-ANT) is not suggested.

Whole network sustainability while reducing the flight capacities is positive and means smaller network is more sustainable but the most important vision of all Airlines is to progress, so in this certain example, increasing total capacity of network by 1.6 is suggested. (Fig. 4).

VI. CONCLUSION

In this paper we looked from different perspective and for the first time airline leg and flight assignment, evaluated from modified leg cost function for airlines. We tried to assess the behavior of legs and network via variation of leg capacities; besides, in this article we tried to identify critical legs via changing practical capacities of airline network components, also we demonstrated how to capture the robustness of airline network in the case of variable legs represented by decreasing and increasing capacities. We used Relative Total Cost indices to assess airline network sustainability in the case of behavior associated with User- Optimization. In this article from different point of view, passenger's rout preference behaviors were the main subject. Future work will use Relative Total Cost Indices to evaluate the robustness of an Airline network considering both U-O and S-O.

APPENDIX

Appendix A is a C++ coding, Appendix B contains the flight information of the certain airline. One can reach this information via email from the first author.

ACKNOWLEDGMENT

The first author is full of gratitude to the Department of Industrial Engineering of Gazi University.

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